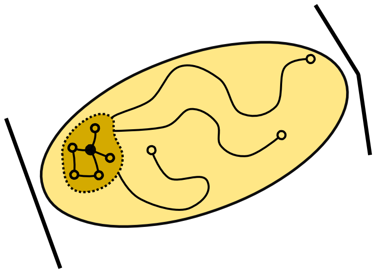
Growing Seed Sets: QW Sampling and st-Connectivity



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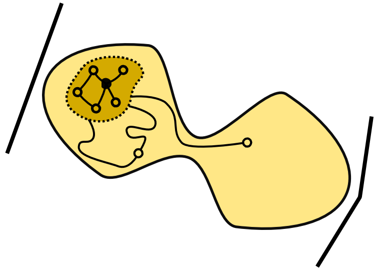

ESA – September 9, 2019 – München

Classical vs Quantum Sampling

Classical sample:

- probability distribution $p \in \mathbb{R}^n$
- normalization $\|p\|_1 = 1$
- π = uniform distribution

Classical vs Quantum Sampling

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- normalization $\|p\|_1 = 1$
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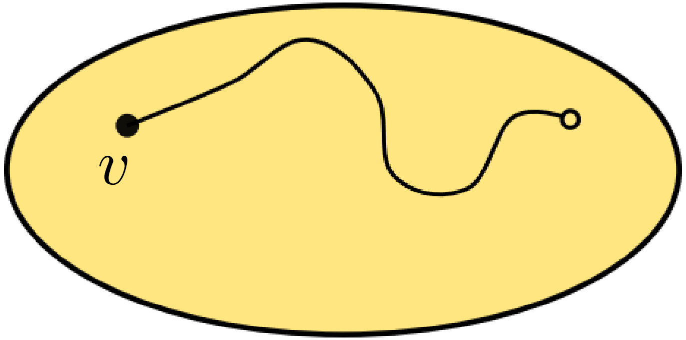
Quantum sample:

- quantum state $|q\rangle \in \mathbb{C}^n$
- normalization $\||q\rangle\|_2 = 1$
- $|\pi\rangle$ = uniform superposition

Random Walk Sampling

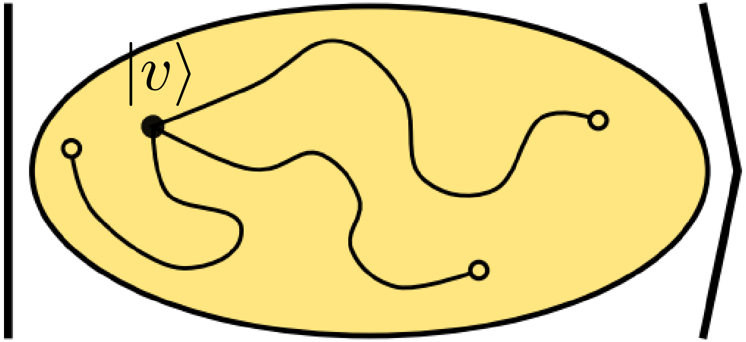
- initial node v, t-step RW $P^t v$
- stationary distribution π and $P^t v \to \pi$
- many applications in graph problems, statistical physics, ...



** throughout talk, assume graphs are regular, bounded degree expanders*

Quantum Walk Sampling

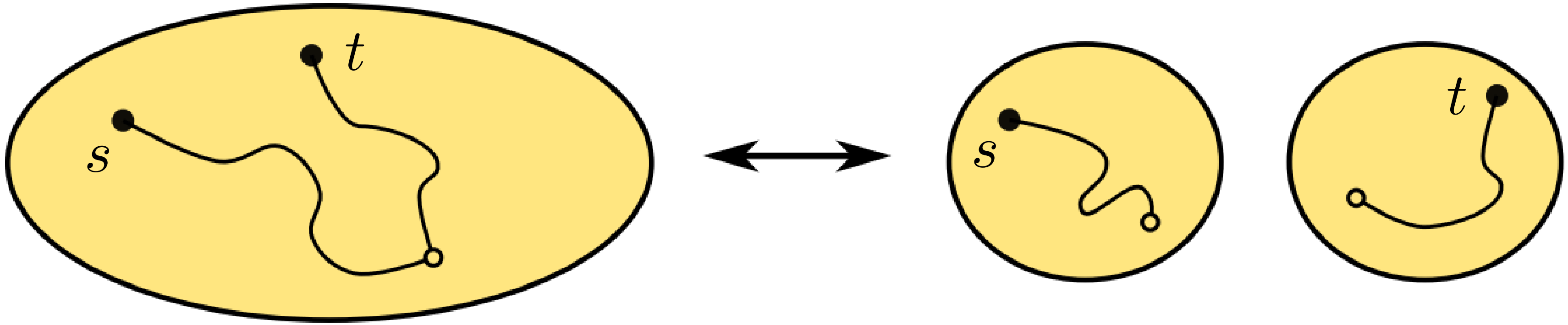
- quantum sample of RW distribution: $|P^t v\rangle = P^t|v\rangle / \|P^t|v\rangle\|$
- use QWs to create $|P^t v\rangle \to |\pi\rangle$. Complexity?
- many applications in element distinctness, formula evaluation, ...



st-Connectivity

Classical: $O(n^{1/2})$

- use RWs from s and t to sample $\pi^{(s)}$ and $\pi^{(t)}$
- look for collision using $O(n^{1/2})$ samples



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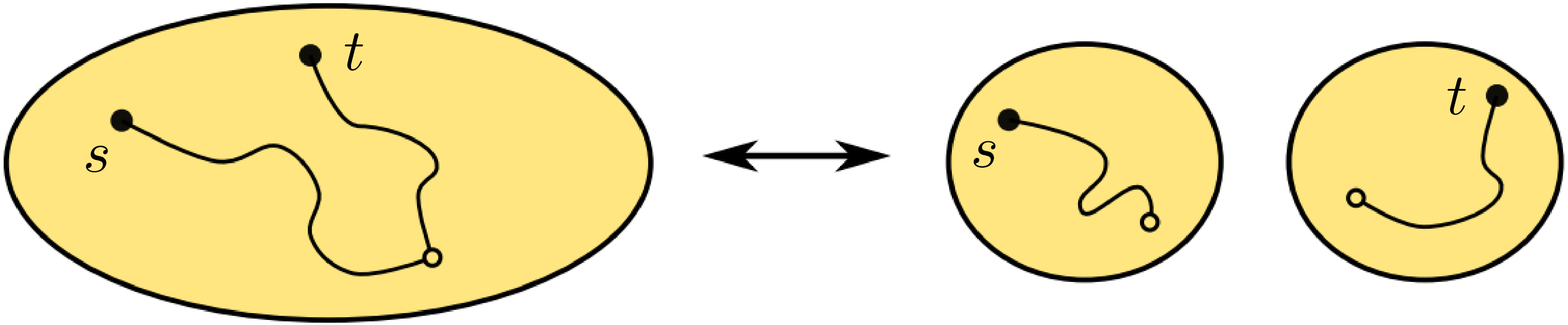

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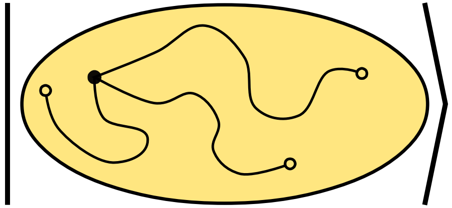

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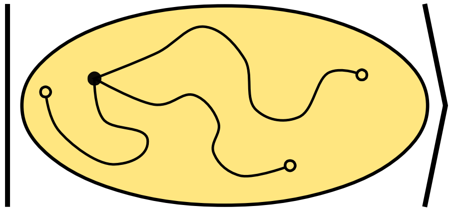
complexity of generating
quantum sample $|\pi\rangle$

Quantum Sampling



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folklore scheme: [Watrous'98]

- t-step QW creates *subnormalized* quantum sample $P^t|v\rangle + |\Gamma\rangle$

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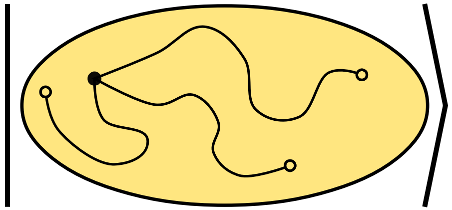

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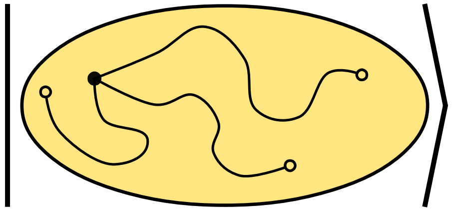

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$\rightarrow$ complexity: $O(\|P^t|v\rangle\|^{-1}) \in O(n^{1/2})$ copies of $P^t|v\rangle + |\Gamma\rangle$

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Quantum: $O(n^{1/2})$

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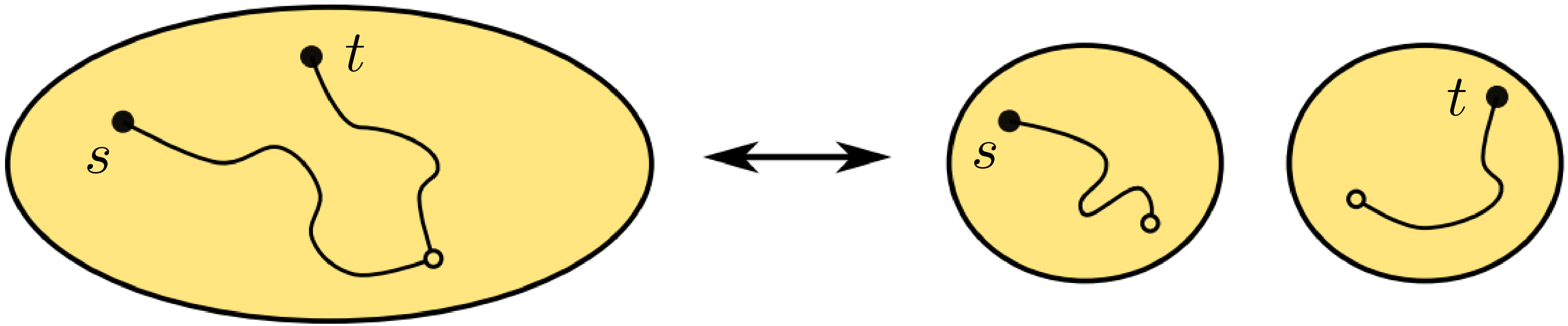

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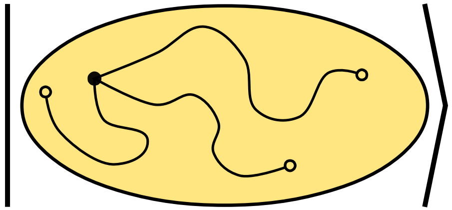

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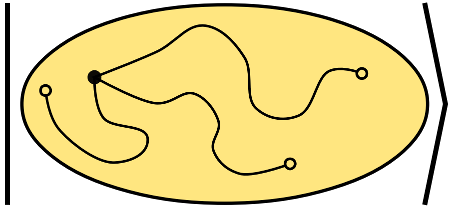
main bottleneck of
folklore approach:

number of samples

$$\|P^t|v\rangle\|^{-1} \approx |\langle\pi|v\rangle|^{-1} \approx n^{1/2}$$

Quantum Sampling



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improve projection on $|\pi\rangle$
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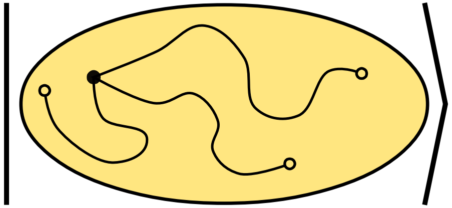
Quantum Sampling



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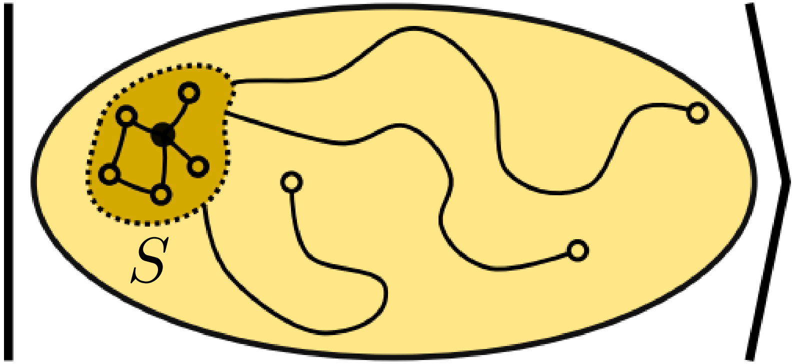

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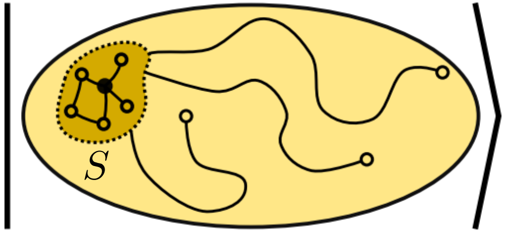
$$S \subset \mathcal{V},\ |S| = n^{1/3}:$$

$$\|P^t|\pi_S\rangle\| \geq |\langle\pi|\pi_S\rangle| \geq n^{-1/3}$$

Quantum Sampling



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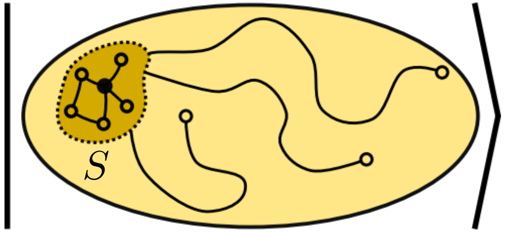
complexity of generating quantum sample $|P^t v\rangle$

improved scheme:

- classically "grow" seed set S of size $n^{1/3}$

Quantum Sampling



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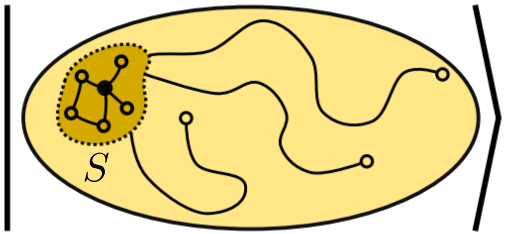

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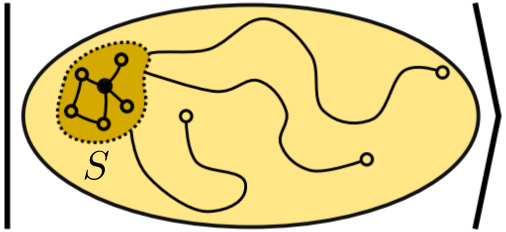

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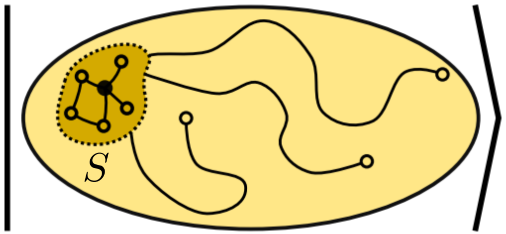

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$\boxed{\text{quantum sample } |\pi\rangle \text{ in } O(n^{1/3}) \text{ QW steps}}$

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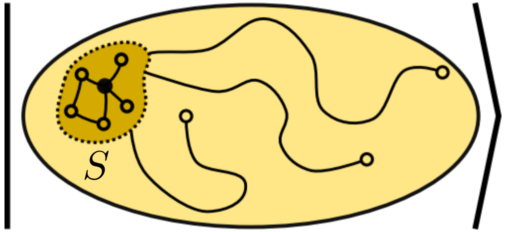

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- use "a … $s\rangle\|$

complexity: …

quantum sample $|\pi\rangle$ in $O(n^{1/3})$ QW steps

space-time tradeoff: exchange $(1, n^{1/2})$ for $(n^{1/3}, n^{1/3})$

st-Connectivity

Classical: $O(n^{1/2})$

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Quantum: $O(n^{1/3})$

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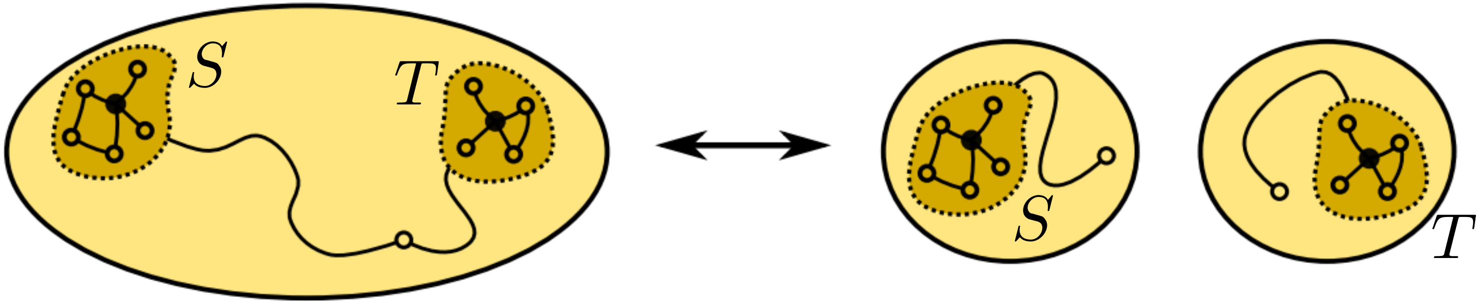
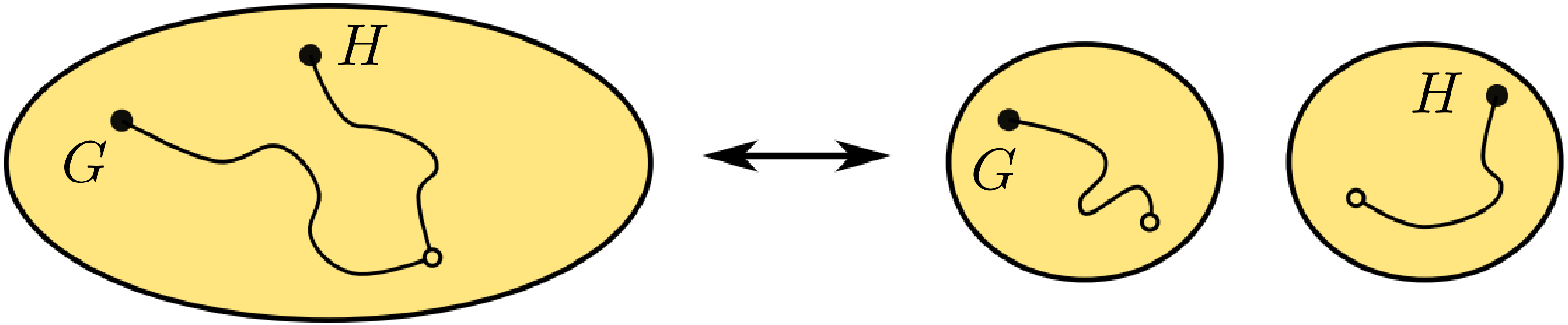


Graph Isomorphism

instance of st-connectivity:
pairwise permutations describe RW over isomorphisms

$$G \to G' \to G'' \to \dots$$

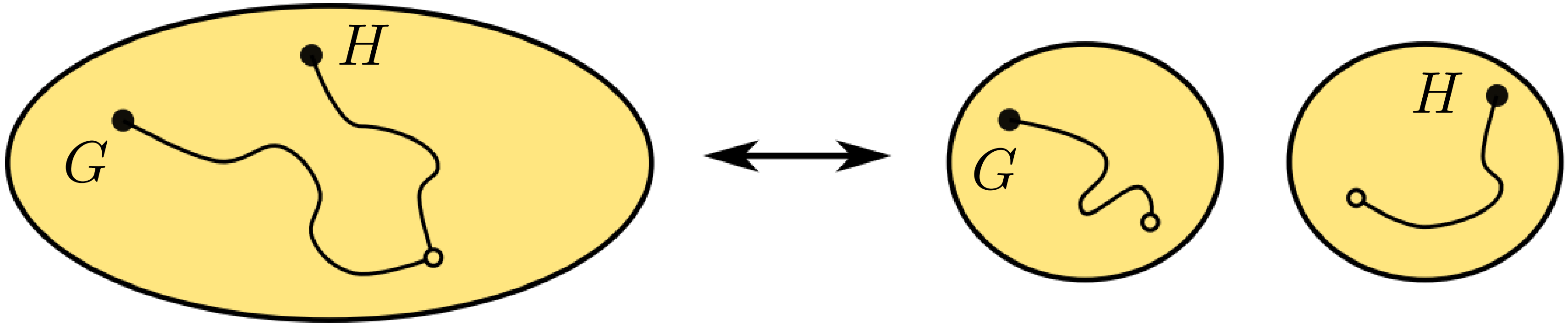
? is there permutation that turns G into H?



Graph Isomorphism

quantum approach:

1. create superpositions $|G\rangle, |H\rangle$ over isomorphisms of G, H
2. compare states using swap test



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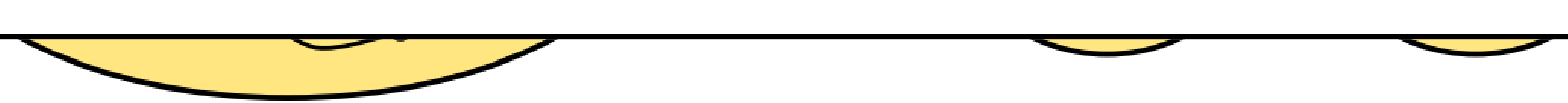
- [AMRR'11]: $\Omega(2^{n/2})$ lower bound for generalized approach



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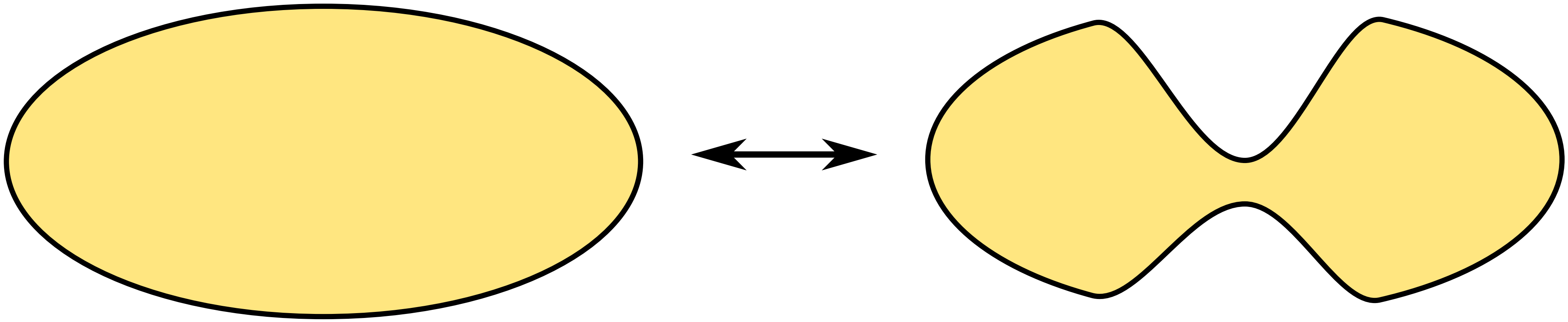
- this work: $O(2^{n/3})$ using QW sampling

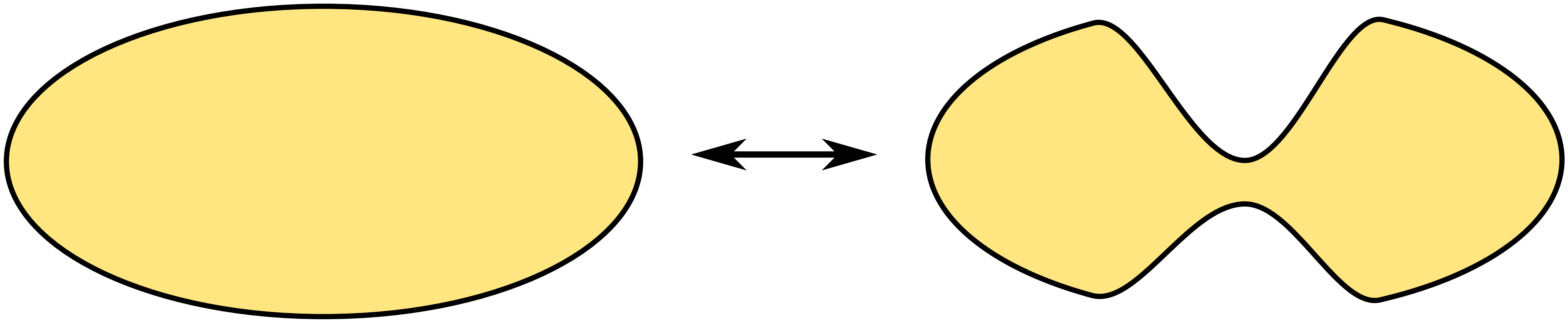
Expansion Testing



[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$, or is G far from any such graph?

Expansion Testing



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does G have expansion $\geq \Upsilon$, or is G far from any such graph?

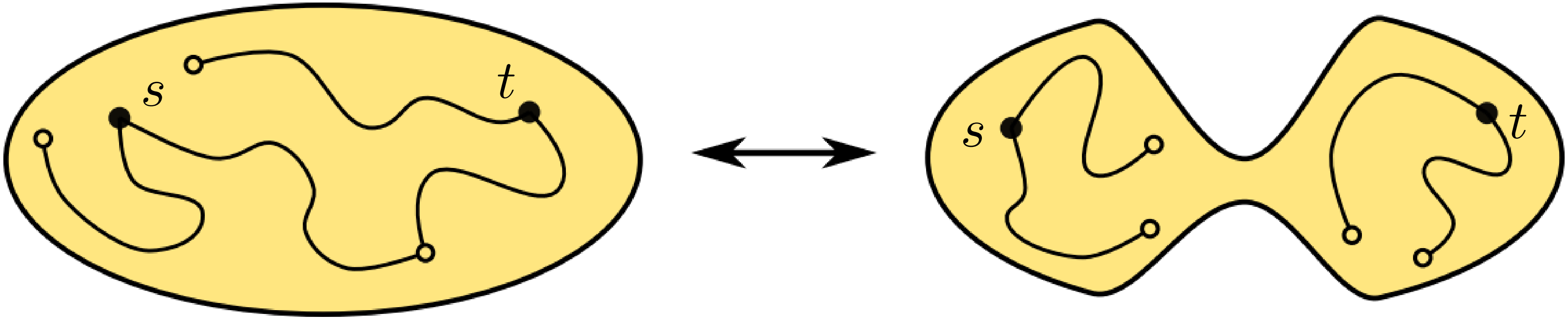
= *robust connectivity*

Expansion Testing

GR expansion tester:

- pick uniformly random nodes s, t
- perform RWs of length Υ^{-2}
- count collisions between $O(n^{1/2})$ samples

if none, `reject`; otherwise, `accept`



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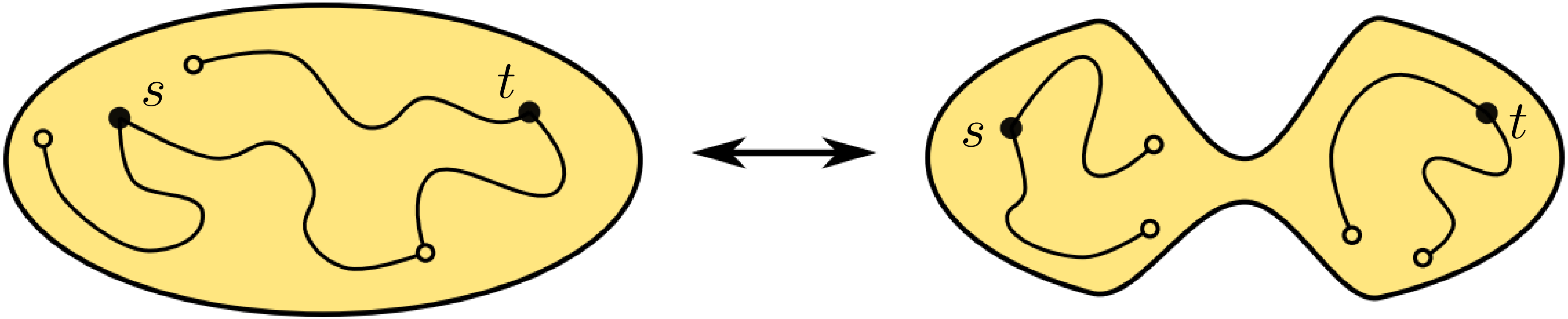

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quantum expansion tester:
[arXiv:1907.02369]

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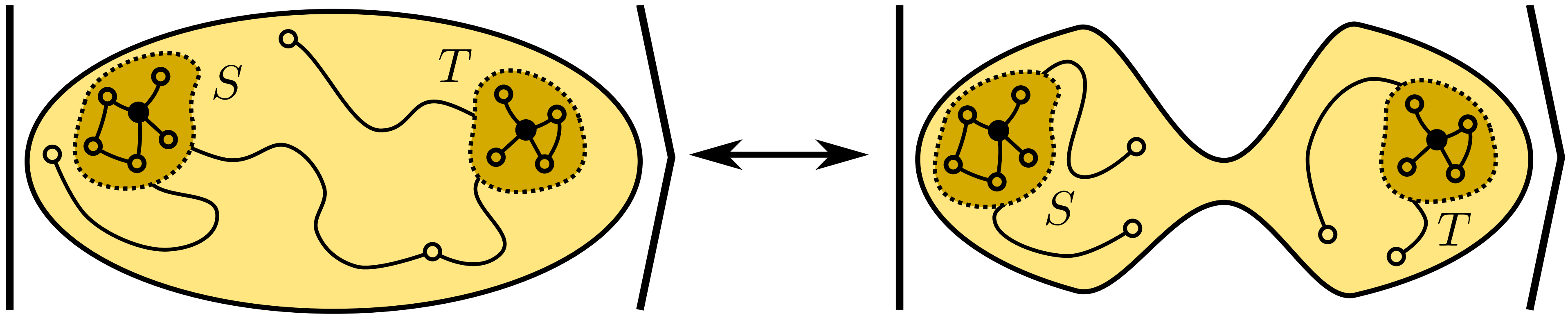

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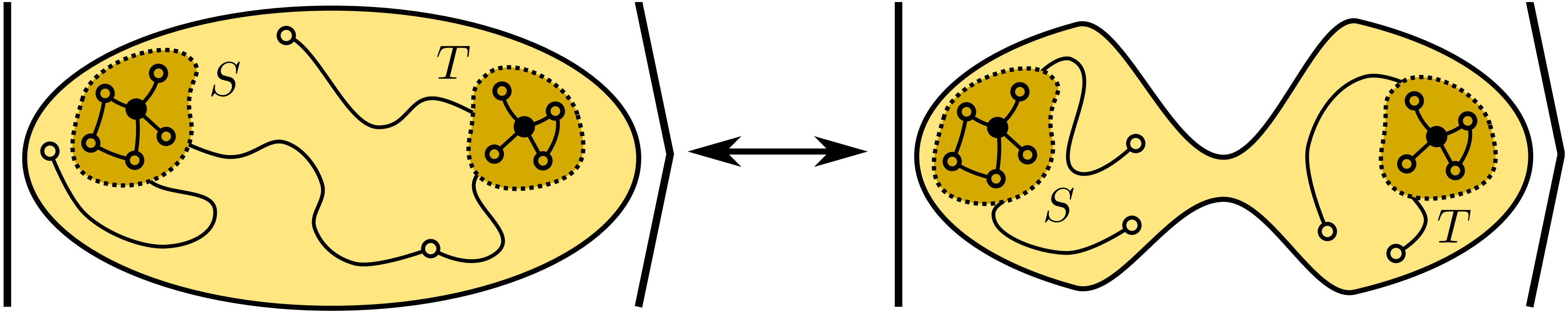

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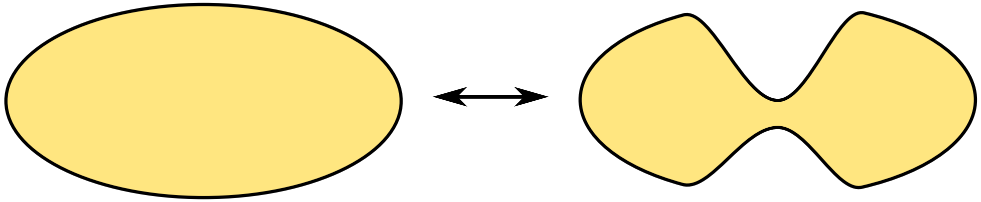

[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets