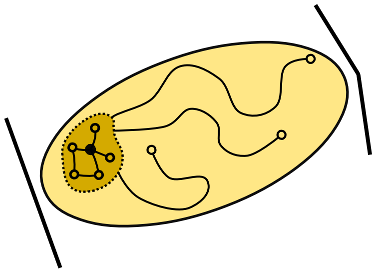
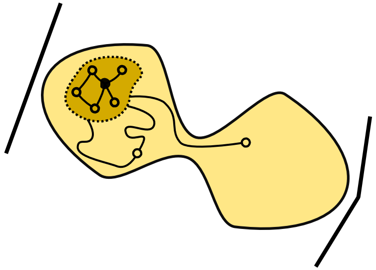
Quantum Expansion Testing using QFF and seed sets



Simon Apers

Inria, CWI

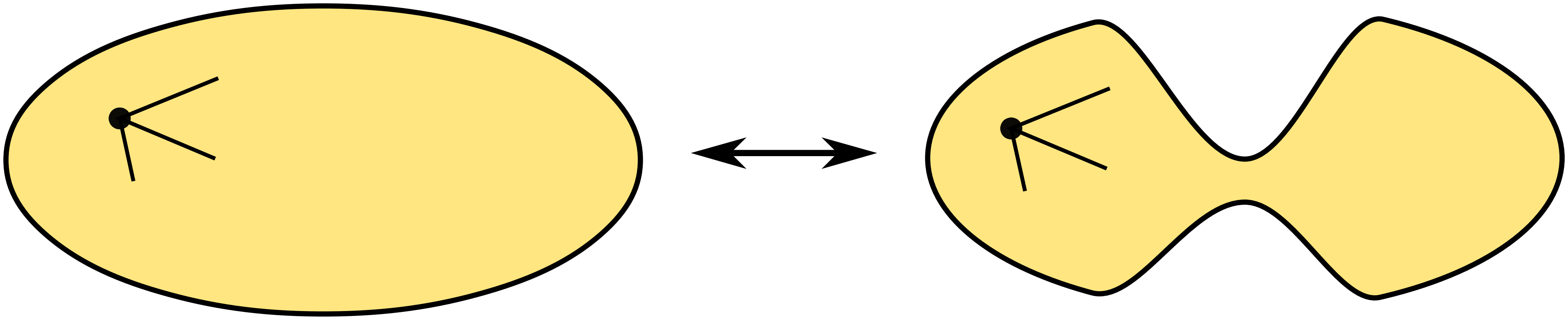
simon.apers@inria.fr



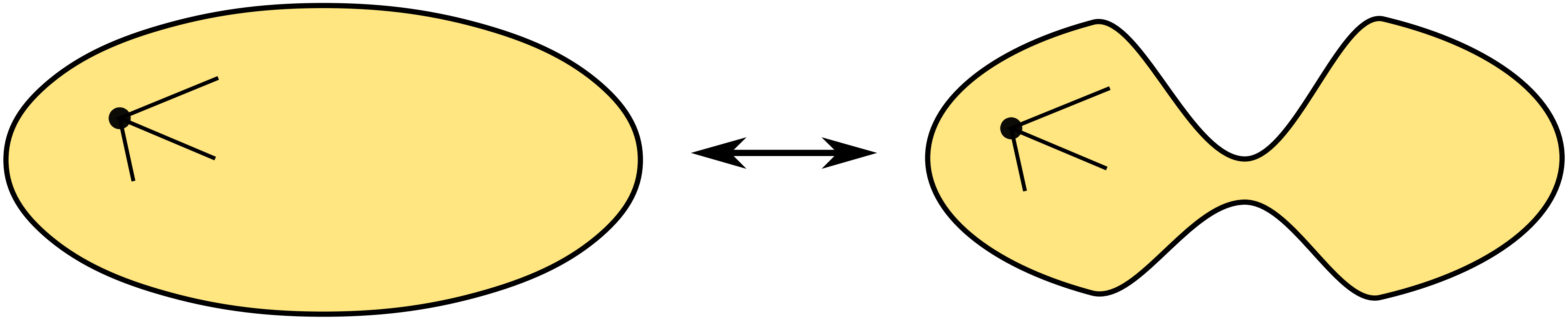
[arXiv:1804.02321 (with A. Sarlette), 1904.11446]

QuIC – June 14, 2019 - Brussels

Expansion Testing

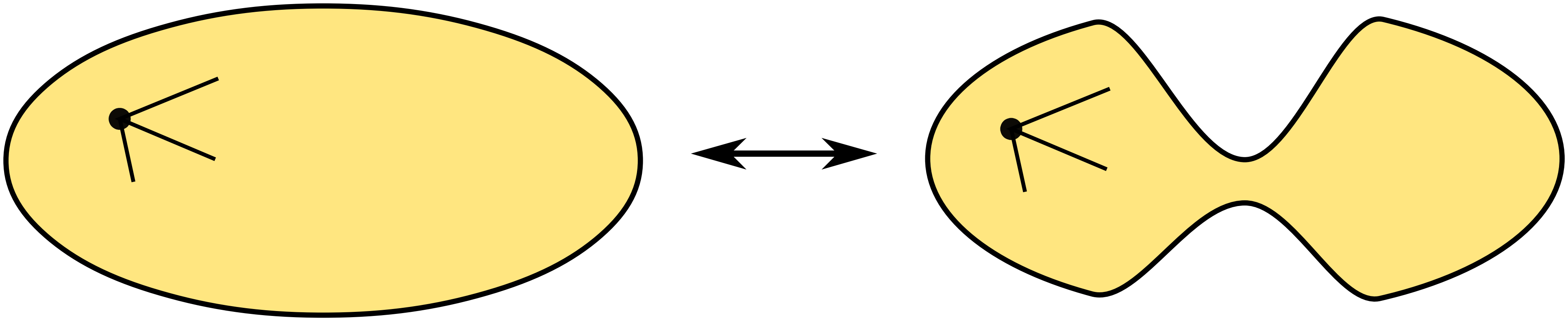


Expansion Testing



$G = (\mathcal{V}, \mathcal{E}) \qquad |\mathcal{V}| = n \qquad |\mathcal{E}| = m$

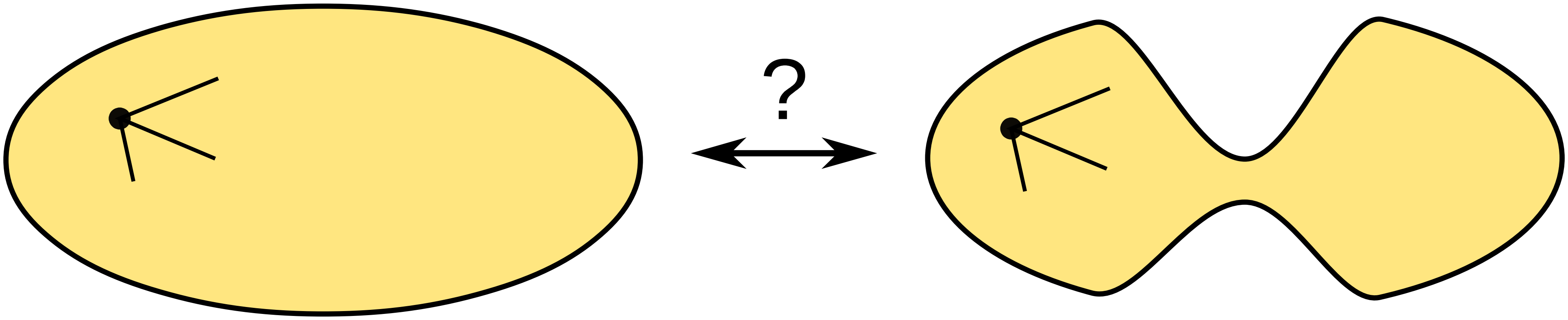
Expansion Testing



$G = (\mathcal{V}, \mathcal{E})$ $\qquad |\mathcal{V}| = n$ $\qquad |\mathcal{E}| = m$

$$\boxed{\Phi = \min_{\mathcal{S} \subset \mathcal{V} : |\mathcal{S}| \leq n/2} |\partial \mathcal{S}| / |\mathcal{S}|}$$

Expansion Testing



[Goldreich-Ron'97]:

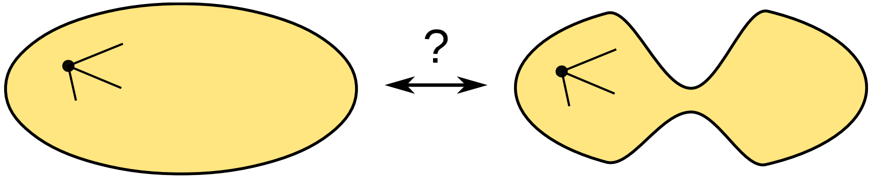
does G have expansion $\geq \Upsilon$, or is G far from any such graph?

Expansion Testing

[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$,

or is G far from any such graph?



[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

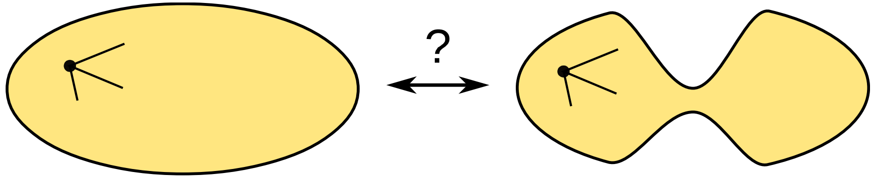
* throughout this talk, we hide polynomial dependency on $\log n$, max degree d, distance parameter ϵ

Expansion Testing

[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$,

or is G far from any such graph?



[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

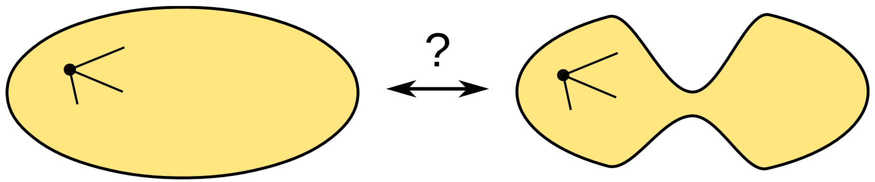
* throughout this talk, we hide polynomial dependency on $\log n$, max degree d, distance parameter ϵ

Expansion Testing

[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$,

or is G far from any such graph?



[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

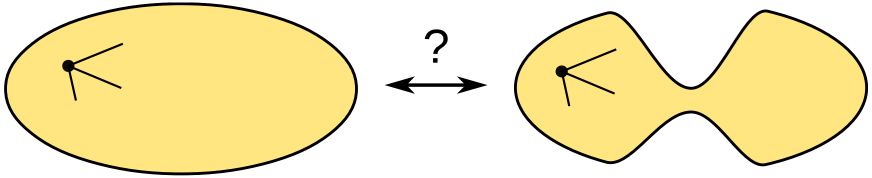
* throughout this talk, we hide polynomial dependency on $\log n$, max degree d, distance parameter ϵ

Expansion Testing

[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$,

or is G far from any such graph?



[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

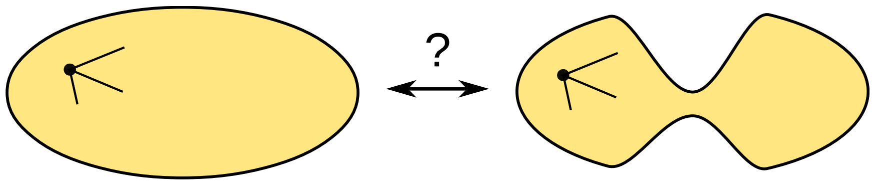
* throughout this talk, we hide polynomial dependency on $\log n$, max degree d, distance parameter ϵ

Expansion Testing

[Goldreich-Ron'97]:

does G have expansion $\geq \Upsilon$,

or is G far from any such graph?



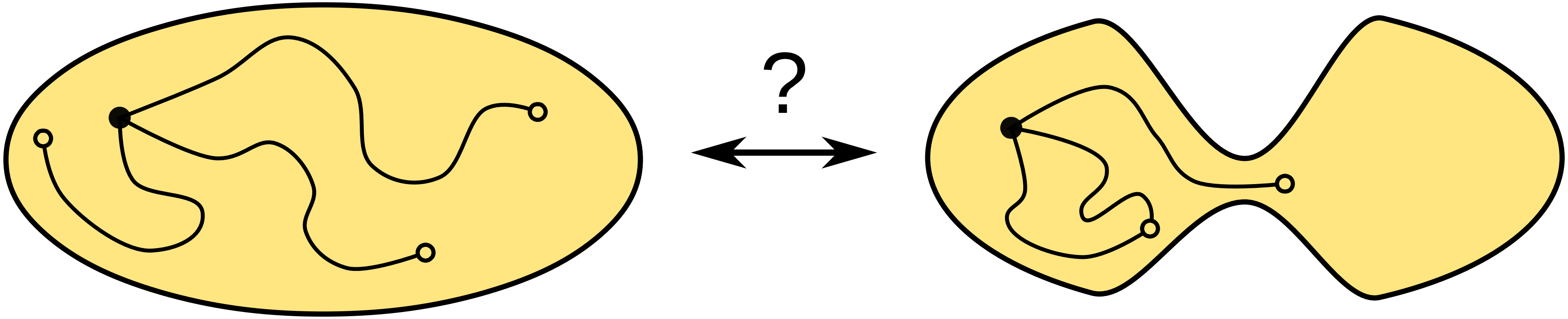
[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	prove conjecture
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

* throughout this talk, we hide polynomial dependency on $\log n$, max degree d, distance parameter ϵ

Expansion Testing

GR expansion tester:

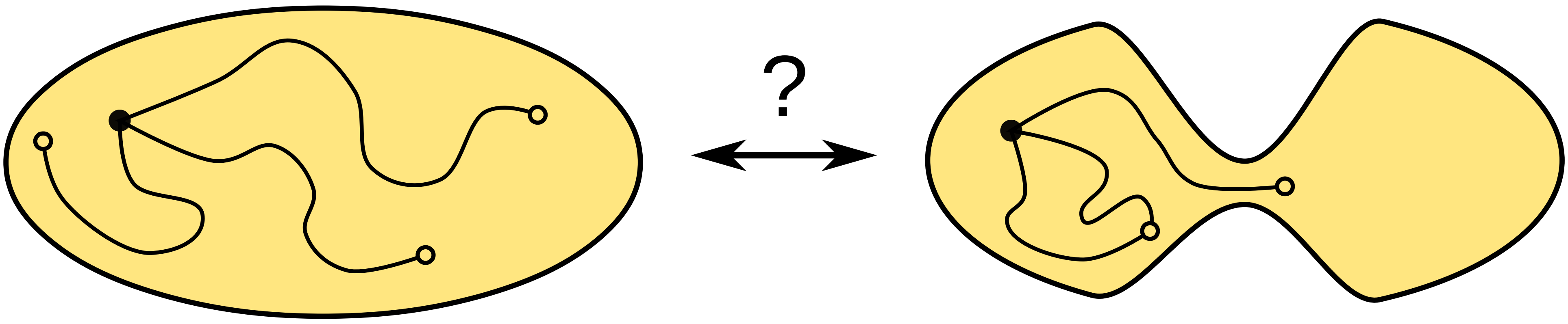
- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints



Expansion Testing

GR expansion tester:

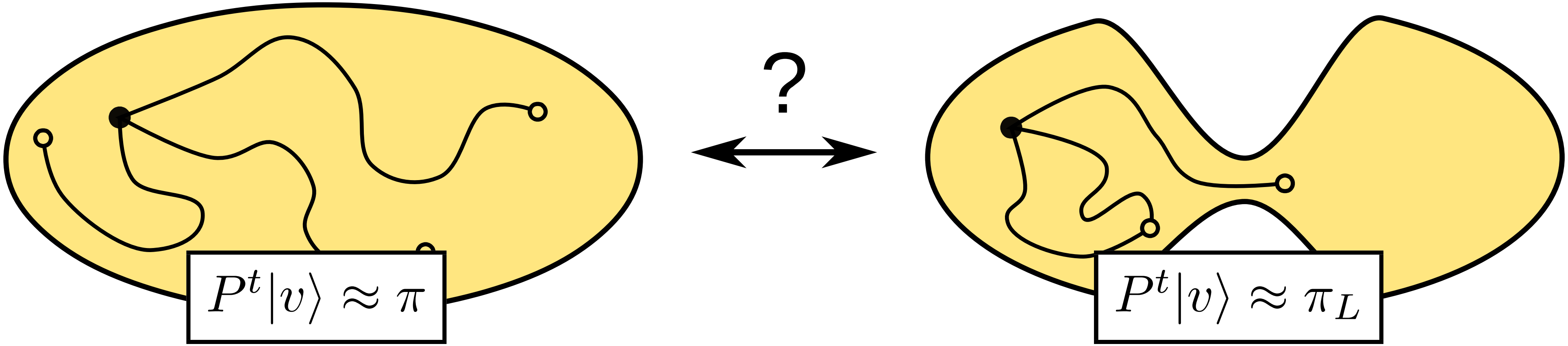
- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints



Expansion Testing

GR expansion tester:

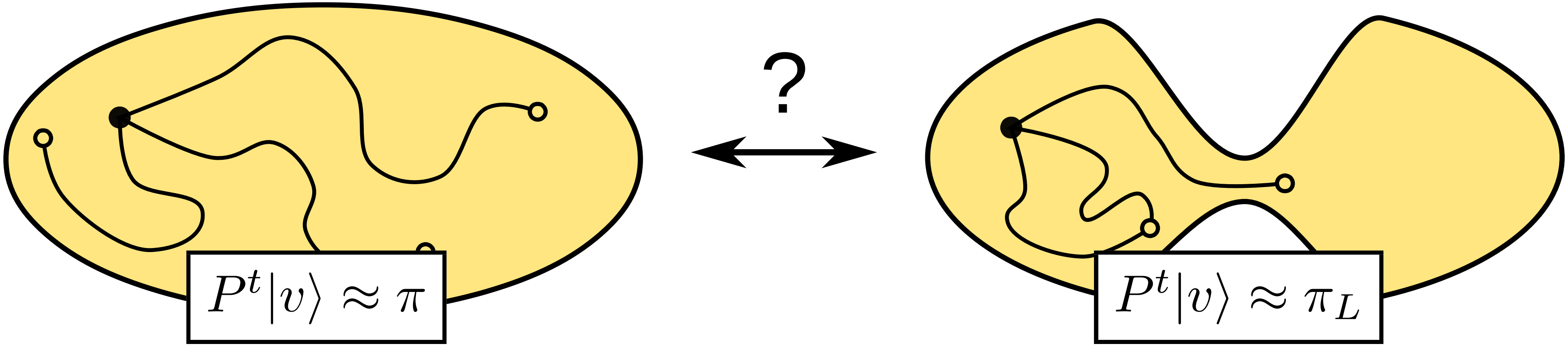
- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints



Expansion Testing

GR expansion tester:

- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints

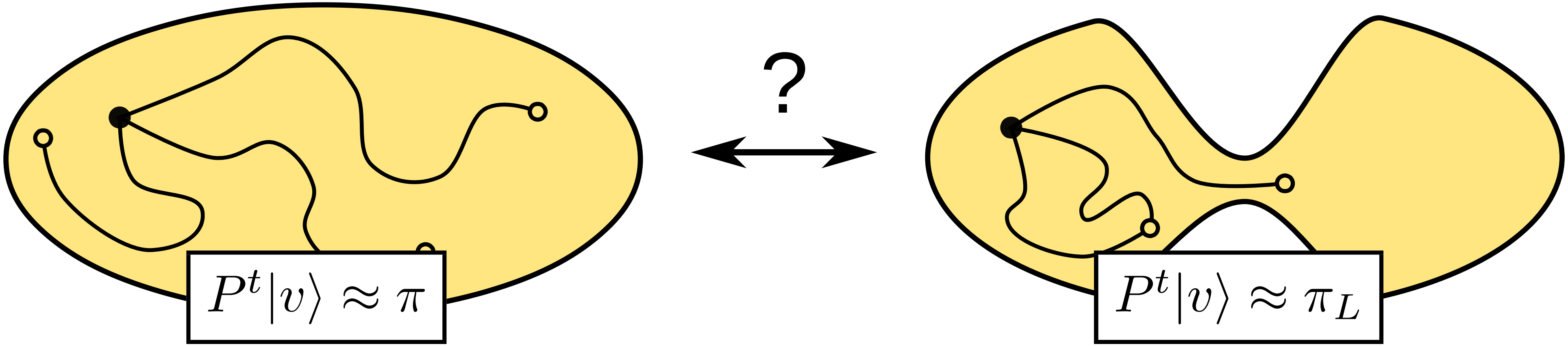


Expansion Testing

GR expansion tester:

- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints

if $> c$, `reject`; otherwise, `accept`



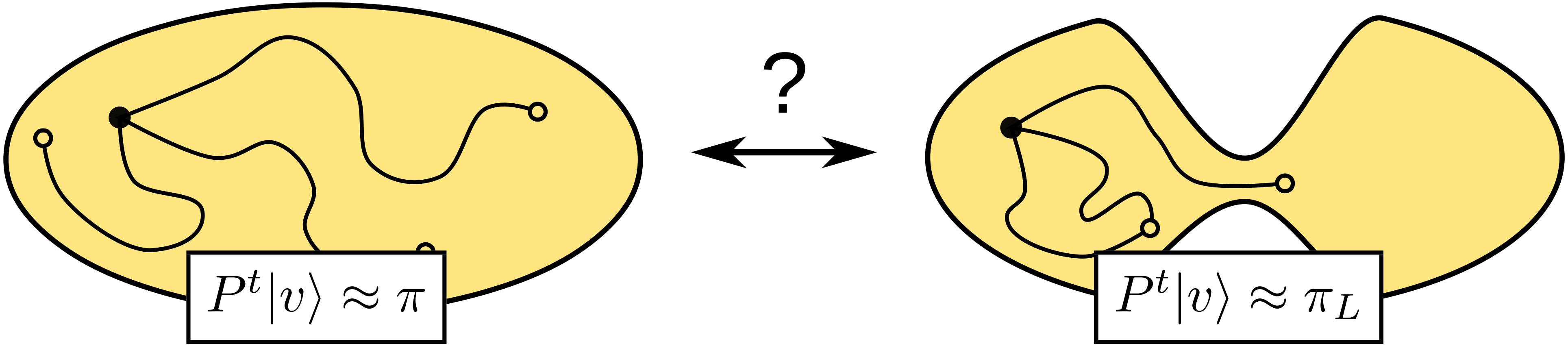
Expansion Testing

GR expansion tester:

$O(n^{1/2}\Upsilon^{-2})$

- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints

if $> c$, $\texttt{reject}$; otherwise, $\texttt{accept}$

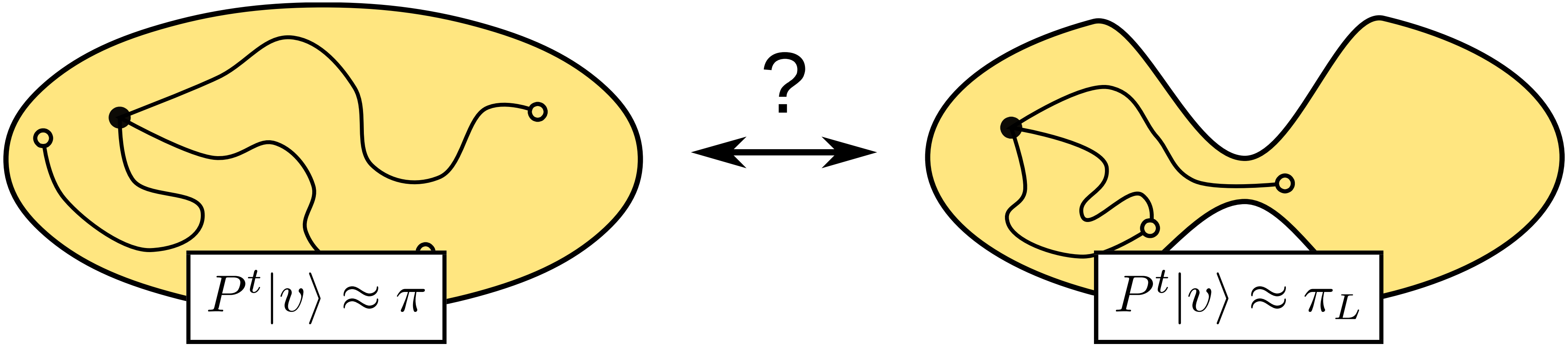


Expansion Testing

GR expansion tester: $O(n^{1/2}\Upsilon^{-2})$

- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints

[ACL '10] speedup to $O(n^{1/3}\Upsilon^{-2})$ using q.algorithm for element distinctness

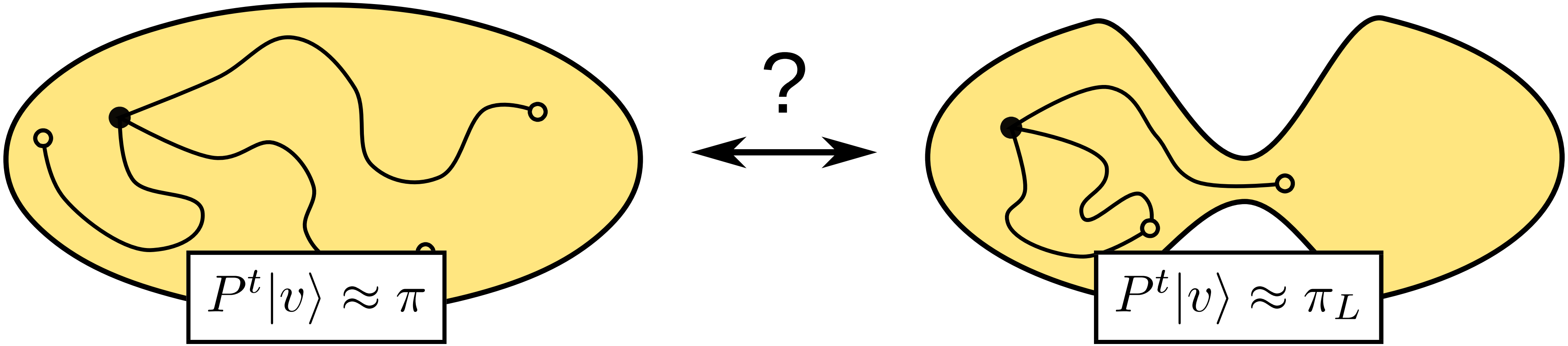


Expansion Testing

GR expansion tester:

- pick uniformly random node v
- perform $n^{1/2}$ RWs of length $t = \Upsilon^{-2}$
- count collisions between endpoints

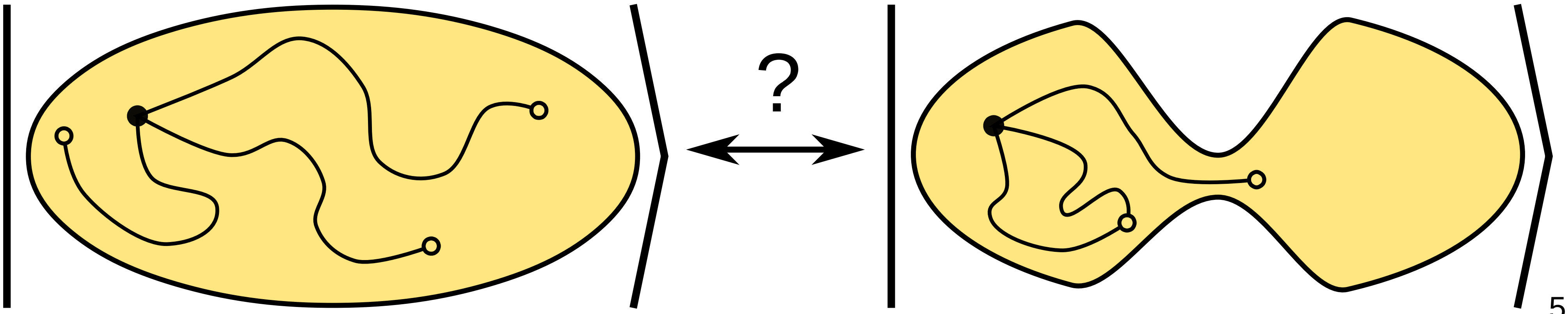
= estimating 2-norm RW distribution $\|P^t|v\rangle\|$



Expansion Testing

quantum expansion tester:

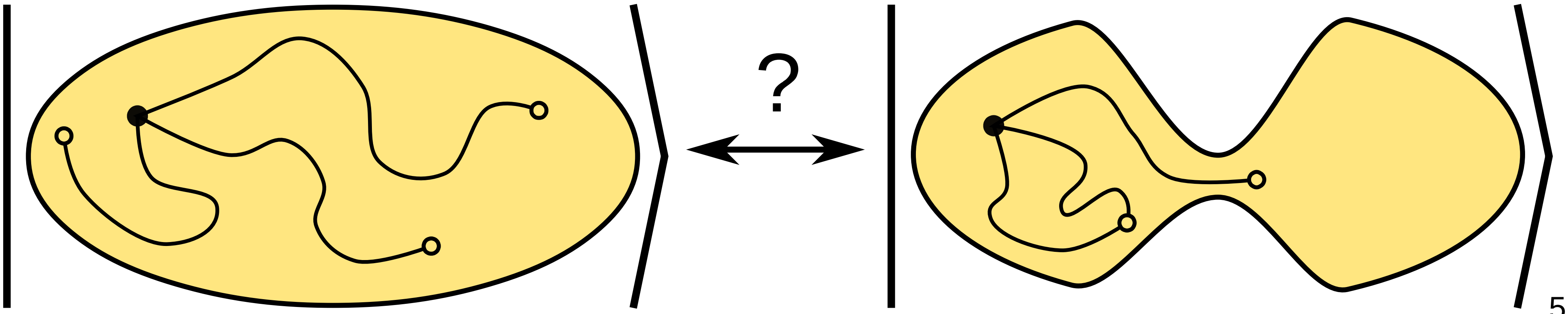
- pick uniformly random node v
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation



Expansion Testing

quantum expansion tester:

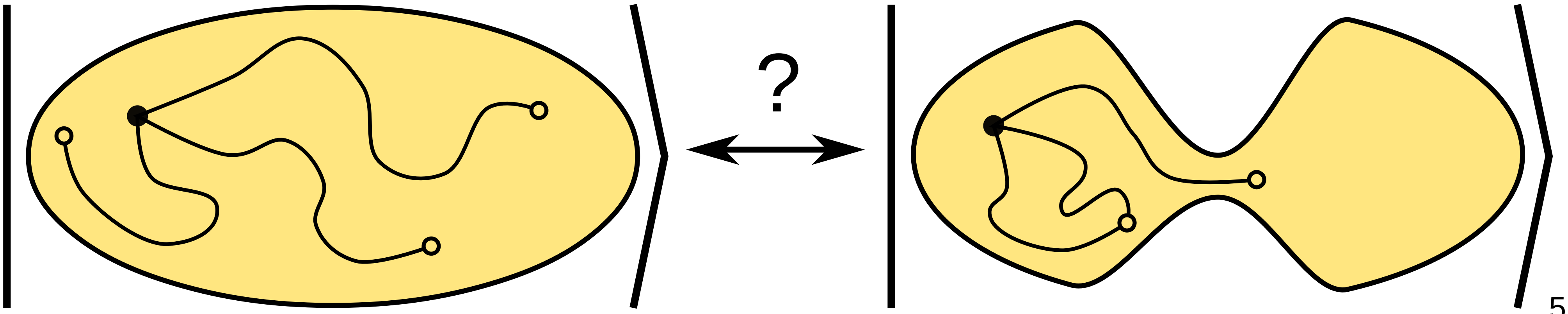
- pick uniformly random node v
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation



Expansion Testing

quantum expansion tester:

- pick uniformly random node v
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation

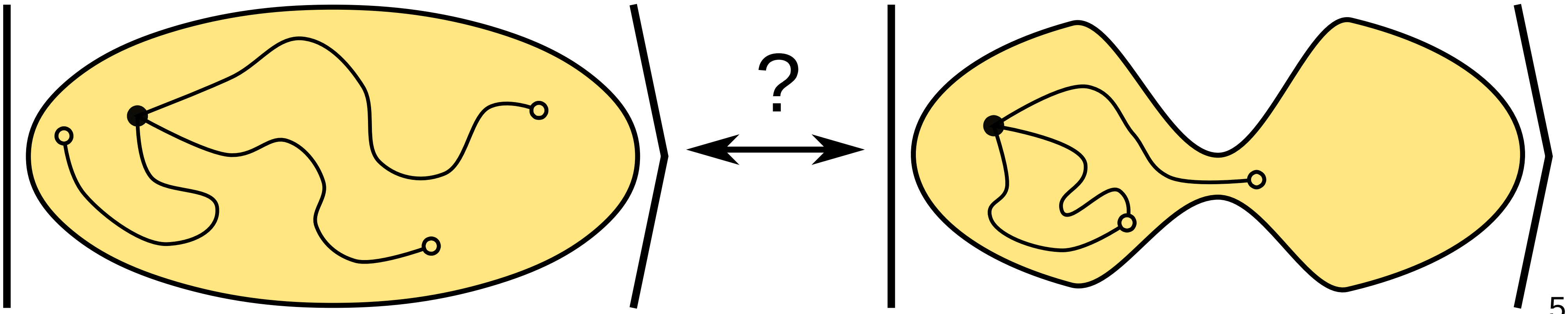


Expansion Testing

quantum expansion tester:

- pick uniformly random node v
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation

if $> (1+c)n^{-1/2}$, `reject`; otherwise, `accept`



Expansion Testing

- pick uniformly random node v

quantum expansion tester:

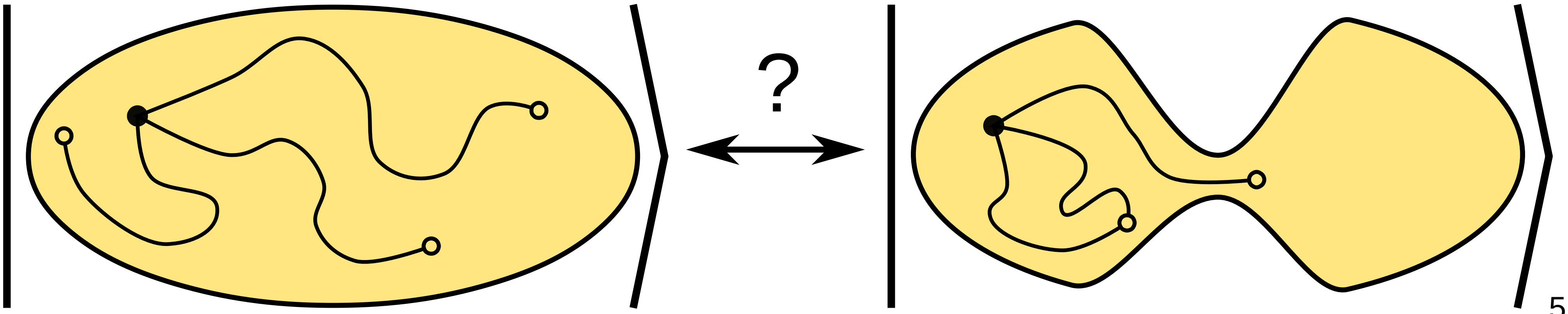
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$

$O(\|P^t|v\rangle\|^{-1}\,\mathrm{QS}_t)$

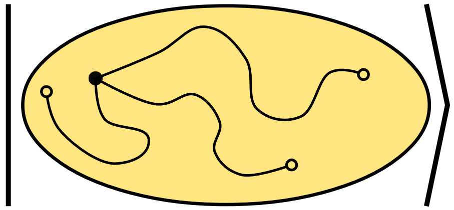
- estimate $\|P^t|v\rangle\|$ via ampl. estimation

$\in O(n^{1/2}\,\mathrm{QS}_t)$

if $> (1+c)n^{-1/2}$, `reject`; otherwise, `accept`



Quantum Sampling



?

complexity QS_t of generating quantum sample $P^t|v\rangle + |\Gamma\rangle$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

prop.: create $P^t|v\rangle + |\Gamma\rangle$ using $O(t)$ QW steps

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

prop.: create $P^t|v\rangle + |\Gamma\rangle$ using $O(t)$ QW steps

✓ for arbitrary t

✗ no speedup

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

prop.: create $P^t|v\rangle + |\Gamma\rangle$ using $O(t)$ QW steps

✓ for arbitrary t

✗ no speedup

equivalently, $\Pi_{\mathcal{V}} U \Pi_{\mathcal{V}} = P \rightarrow$ (first?) block encoding!

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } P^t|v\rangle + |\Gamma\rangle \text{ using } O(t) \text{ QW steps}}$

✓ for arbitrary t

✗ no speedup

- [Amb'03], [Sz'04], [MNRS'06]: $W = (2\Pi_{\mathcal{V}} - I)U$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } P^t|v\rangle + |\Gamma\rangle \text{ using } O(t) \text{ QW steps}}$

✓ for arbitrary t

✗ no speedup

- [Amb'03], [Sz'04], [MNRS'06]: $W = (2\Pi_{\mathcal{V}} - I)U$

$$|v\rangle \mapsto W^t|v\rangle \quad \text{s.t.} \quad W^t|v\rangle = T_t(P)|v\rangle + |\Gamma\rangle$$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } P^t|v\rangle + |\Gamma\rangle \text{ using } O(t) \text{ QW steps}}$

✓ for arbitrary t

✗ no speedup

- [Amb'03], [Sz'04], [MNRS'06]: $W = (2\Pi_{\mathcal{V}} - I)U$

$$|v\rangle \mapsto W^t|v\rangle \quad \text{s.t.} \quad W^t|v\rangle = T_t(P)|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } \pi + |\Gamma\rangle \text{ using } O(\delta^{-1/2}) \text{ QW steps}}$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } P^t|v\rangle + |\Gamma\rangle \text{ using } O(t) \text{ QW steps}}$

✓ for arbitrary t

✗ no speedup

- [Amb'03], [Sz'04], [MNRS'06]: $W = (2\Pi_{\mathcal{V}} - I)U$

$$|v\rangle \mapsto W^t|v\rangle \quad \text{s.t.} \quad W^t|v\rangle = T_t(P)|v\rangle + |\Gamma\rangle$$

$\boxed{\text{prop.: create } \pi + |\Gamma\rangle \text{ using } O(\delta^{-1/2}) \text{ QW steps}}$

$= \lim_{t\to\infty} P^t|v\rangle$

Quantum Sampling

- [Watrous'98]: (first?) discrete-time QW

$$|v\rangle \mapsto U|v\rangle \quad \text{s.t.} \quad U|v\rangle = P|v\rangle + |\Gamma\rangle$$

prop.: create $P^t|v\rangle + |\Gamma\rangle$ using $O(t)$ QW steps

✓ for arbitrary t

✗ no speedup

- [Amb'03], [Sz'04], [MNRS'06]: $W = (2\Pi_{\mathcal{V}} - I)U$

$$|v\rangle \mapsto W^t|v\rangle \quad \text{s.t.} \quad W^t|v\rangle = T_t(P)|v\rangle + |\Gamma\rangle$$

prop.: create $\pi + |\Gamma\rangle$ using $O(\delta^{-1/2})$ QW steps

$= \lim_{t\to\infty} P^t|v\rangle$

✗ only limit behavior

✓ quadratic speedup

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

classic in spectral graph theory: proves that $\delta \in O(D^{-2} \log^2 n)$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

  - LCU: $|v\rangle \mapsto V_\tau |v\rangle$ s.t. $V_\tau |v\rangle = \sum c_t W^t |v\rangle + |\Gamma\rangle$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1, 1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

  - LCU: $|v\rangle \mapsto V_\tau |v\rangle$ s.t. $V_\tau |v\rangle = \sum c_t W^t |v\rangle + |\Gamma\rangle$

$$= \sum c_t T_t(P)|v\rangle + |\Gamma\rangle$$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

 - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1, 1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

 - LCU: $|v\rangle \mapsto V_\tau |v\rangle$ s.t. $V_\tau |v\rangle = \sum c_t W^t |v\rangle + |\Gamma\rangle$

$$= \sum c_t T_t(P) |v\rangle + |\Gamma\rangle$$

$$= p_\tau(P) |v\rangle + |\Gamma\rangle$$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

  - LCU: $|v\rangle \mapsto V_\tau|v\rangle$ s.t.
$$
\begin{aligned}
V_\tau|v\rangle &= \sum c_t W^t|v\rangle + |\Gamma\rangle \\
&= \sum c_t T_t(P)|v\rangle + |\Gamma\rangle \\
&= p_\tau(P)|v\rangle + |\Gamma\rangle = (P^t + O(\epsilon))|v\rangle + |\Gamma\rangle
\end{aligned}
$$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

  - appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

  - LCU: $|v\rangle \mapsto V_\tau |v\rangle$ s.t.

$$\begin{aligned} V_\tau |v\rangle &= \sum c_t W^t |v\rangle + |\Gamma\rangle \\ &= \sum c_t T_t(P) |v\rangle + |\Gamma\rangle \\ &= p_\tau(P)|v\rangle + |\Gamma\rangle = (P^t + O(\epsilon))|v\rangle + |\Gamma\rangle \end{aligned}$$

$\boxed{\text{prop.: } \epsilon\text{-approximation of } P^t|v\rangle + |\Gamma\rangle \text{ using } O(\sqrt{t \log \epsilon^{-1}}) \text{ QW steps}}$

Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

- appr. lemma: $x^t = p_\tau(x) + O(\epsilon)$ for $x \in [-1,1]$, $\tau \in O(\sqrt{t \log \epsilon^{-1}})$

- LCU: $|v\rangle \mapsto V_\tau |v\rangle$ s.t.

$$\begin{aligned} V_\tau |v\rangle &= \sum c_t W^t |v\rangle + |\Gamma\rangle \\ &= \sum c_t T_t(P)|v\rangle + |\Gamma\rangle \\ &= p_\tau(P)|v\rangle + |\Gamma\rangle = (P^t + O(\epsilon))|v\rangle + |\Gamma\rangle \end{aligned}$$

$$\boxed{\text{prop.: } \epsilon\text{-approximation of } P^t|v\rangle + |\Gamma\rangle \text{ using } O(\sqrt{t \log \epsilon^{-1}}) \text{ QW steps}}$$

✓ for arbitrary t ✓ quadratic speedup

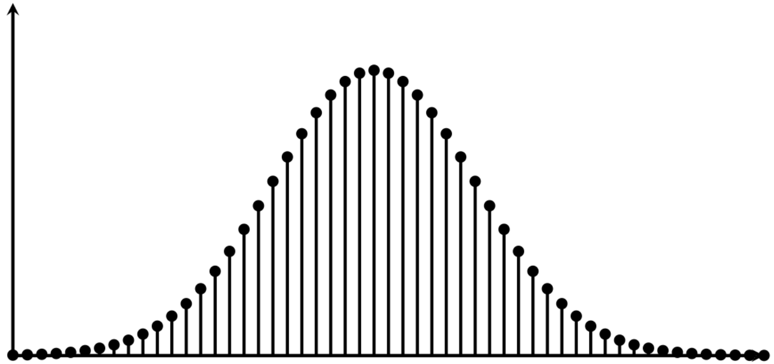
Quantum Fast-Forwarding

- [A-Sarlette'18]: "polish" quadratic speedup

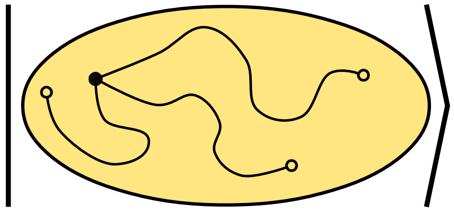
$\boxed{\text{prop.: } \epsilon\text{-approximation of } P^t|v\rangle + |\Gamma\rangle \text{ using } O(\sqrt{t \log \epsilon^{-1}}) \text{ QW steps}}$

✓ for arbitrary t ✓ quadratic speedup

optimal (t, ϵ)-dependency:
RW on the line has ϵ-weight
outside $[-\sqrt{t \log \epsilon^{-1}}, \sqrt{t \log \epsilon^{-1}}]$



Quantum Sampling

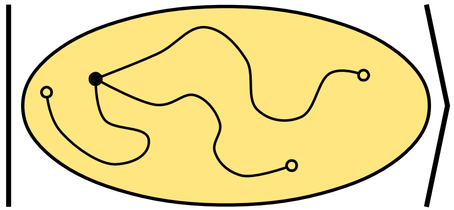


?

complexity QS_t of generating quantum sample $P^t|v\rangle + |\Gamma\rangle$

	QW steps	
[Watrous'98]	$O(t)$	
[Amb'03], [Sz'04], [MNRS'06]	$O(\delta^{-1/2})$	only for $t \to \infty$
[A-Sarlette'18]	$O(t^{1/2})$	quantum fast-forwarding

Quantum Sampling

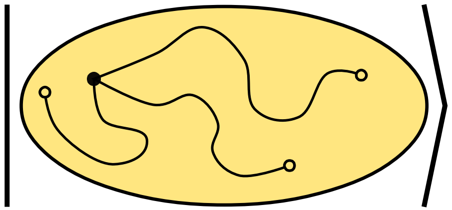


?

complexity QS_t of generating quantum sample $P^t|v\rangle + |\Gamma\rangle$

	QW steps	apps
[Watrous'98]	$O(t)$	graph connectivity
[Amb'03], [Sz'04], [MNRS'06]	$O(\delta^{-1/2})$	QW search, Gibbs sampling
[A-Sarlette'18]	$O(t^{1/2})$	QW search,* prop. testing

Quantum Sampling



?

complexity QS_t of generating quantum sample $P^t|v\rangle + |\Gamma\rangle$

	QW steps	apps
[Watrous'98]	$O(t)$	graph connectivity
[Amb'03], [Sz'04], [MNRS'06]	$O(\delta^{-1/2})$	QW search, Gibbs sampling
[A-Sarlette'18]	$O(t^{1/2})$	QW search,* prop. testing

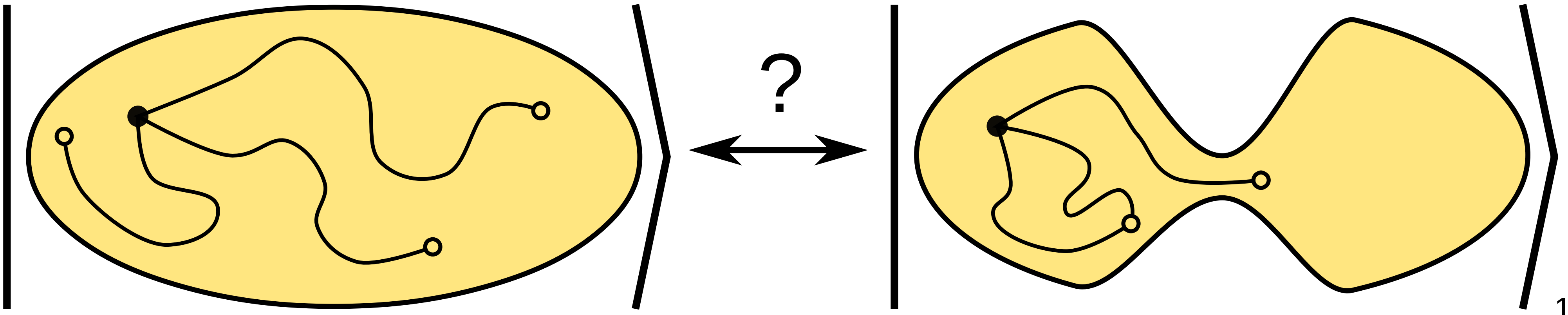
*[Ambainis-Gilyén-Kokainis-Jeffery'19, A-Gilyén-Jeffery'19] use QFF to make progress on open questions in QW search

Expansion Testing

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}\,\mathrm{QS}_t)$$
$$\in O(n^{1/2}\,\mathrm{QS}_t)$$

- pick uniformly random node v
- create quantum sample $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation



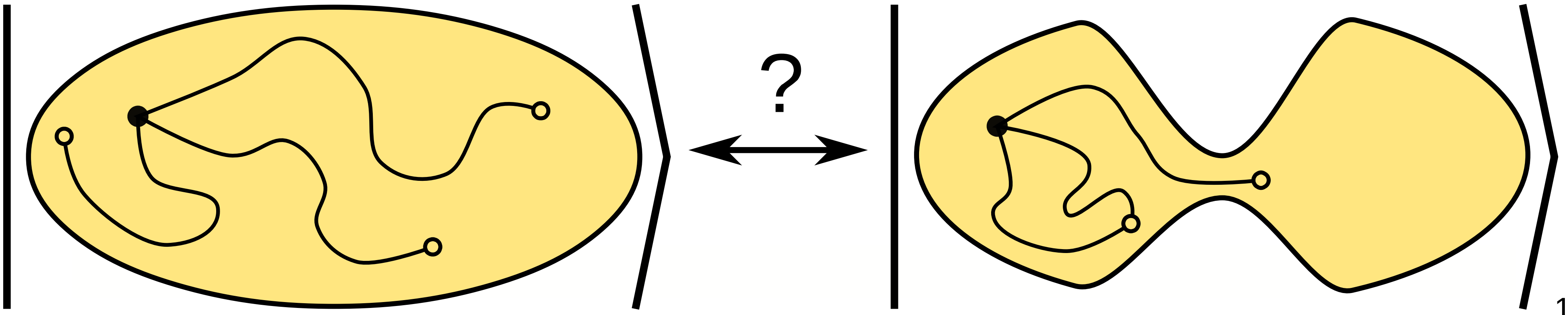
Expansion Testing

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}\,\mathrm{QS}_t)$$

$$\in O(n^{1/2}t^{1/2})$$

- pick uniformly random node v
- use QFF to create $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation



Expansion Testing

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}\,\mathrm{QS}_t)$$

$$\in O(n^{1/2}t^{1/2})$$

- pick uniformly random node v
- use QFF to create $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation

[Goldreich-Ron '00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS '07], [KS '07], [NS '07]	$O(n^{1/2}\Upsilon^{-2})$	
[Ambainis-Childs-Liu '10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette '18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A '19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

Expansion Testing

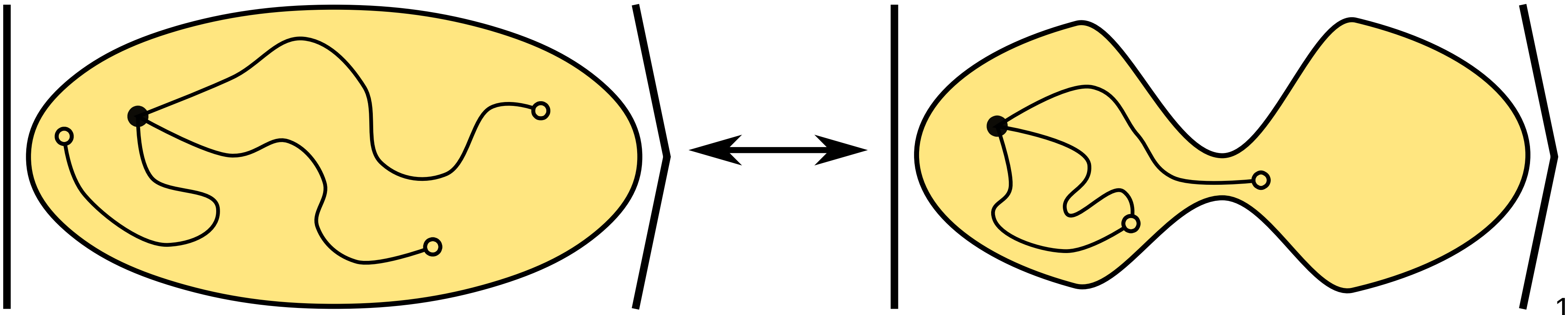
- pick uniformly random node v
- use QFF to create $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$

$$\in O(n^{1/2}t^{1/2})$$

! suboptimal dependency on n

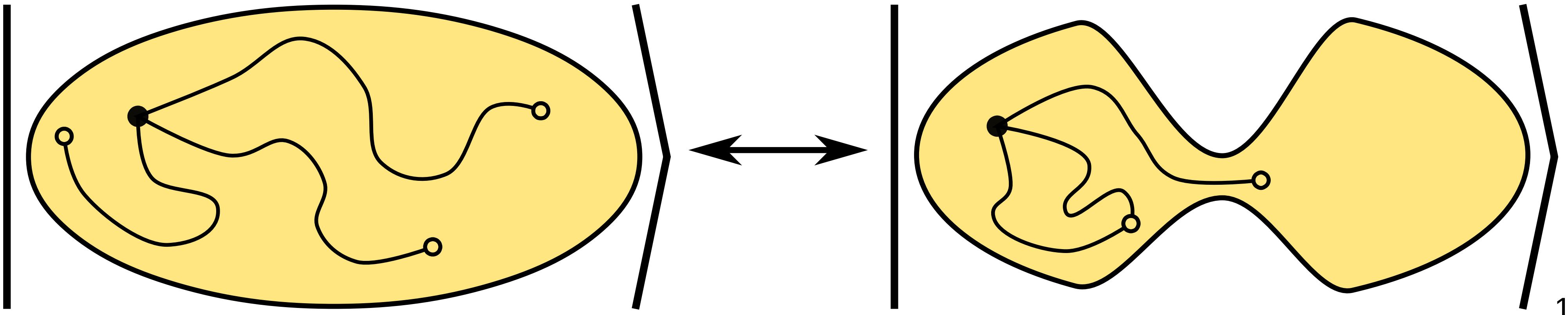


Expansion Testing

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$

$$\in O(n^{1/2}t^{1/2})$$! suboptimal dependency on n



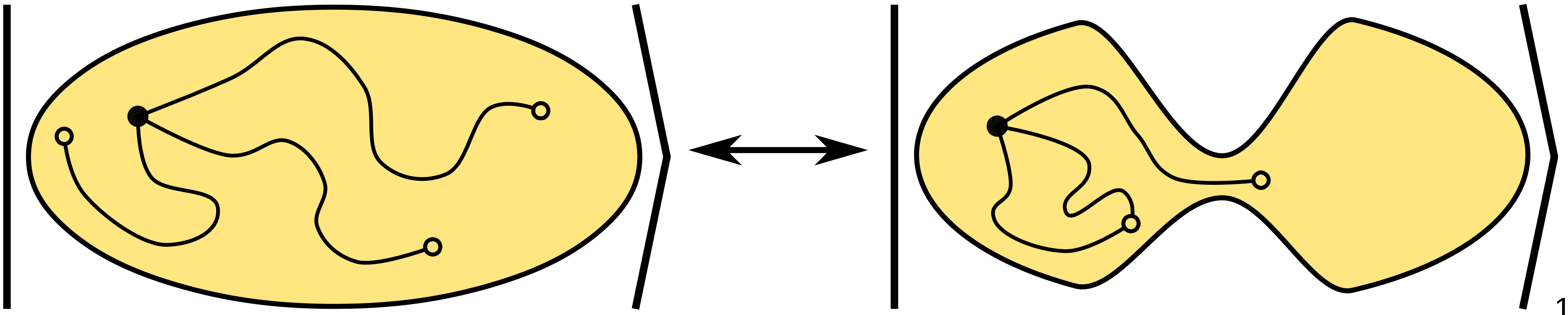
Expansion Testing

$$\|P^t|v\rangle\| \geq |\langle\pi|v\rangle| \geq n^{-1/2}$$

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$

$$\in O(n^{1/2}t^{1/2})$$! suboptimal dependency on n



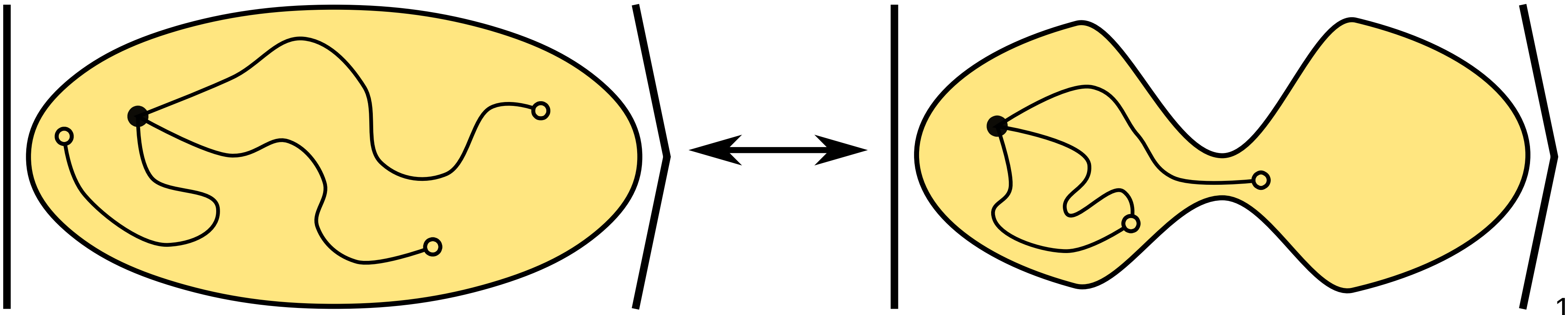
Expansion Testing

$$\|P^t|v\rangle\| \geq |\langle\pi|v\rangle| \geq n^{-1/2}$$

quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/2}t^{1/2})$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



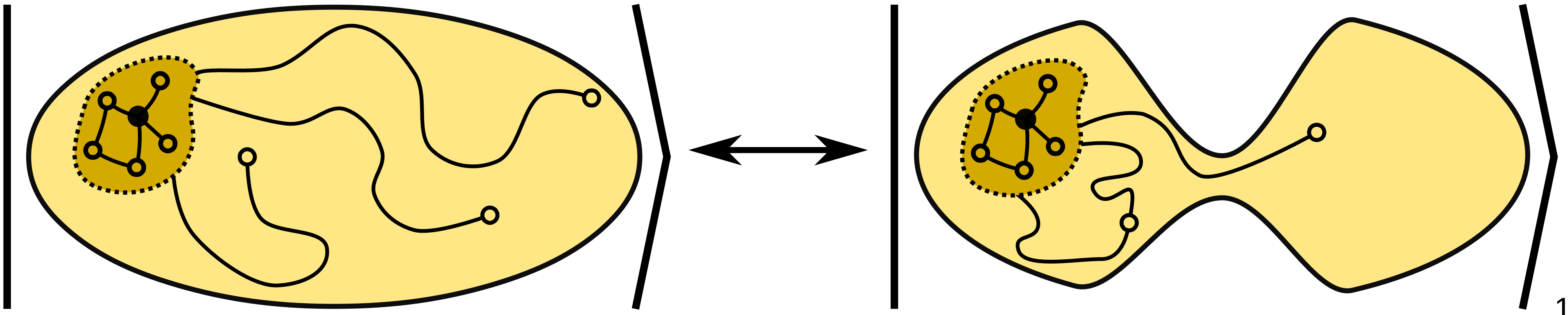
Expansion Testing

$$\| P^t |v\rangle \| \geq |\langle \pi | v \rangle| \geq n^{-1/2}$$

quantum expansion tester:

$$O(\| P^t |v\rangle \|^{-1} t^{1/2})$$
$$\in O(n^{1/2} t^{1/2})$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



Expansion Testing

quantum expansion tester:

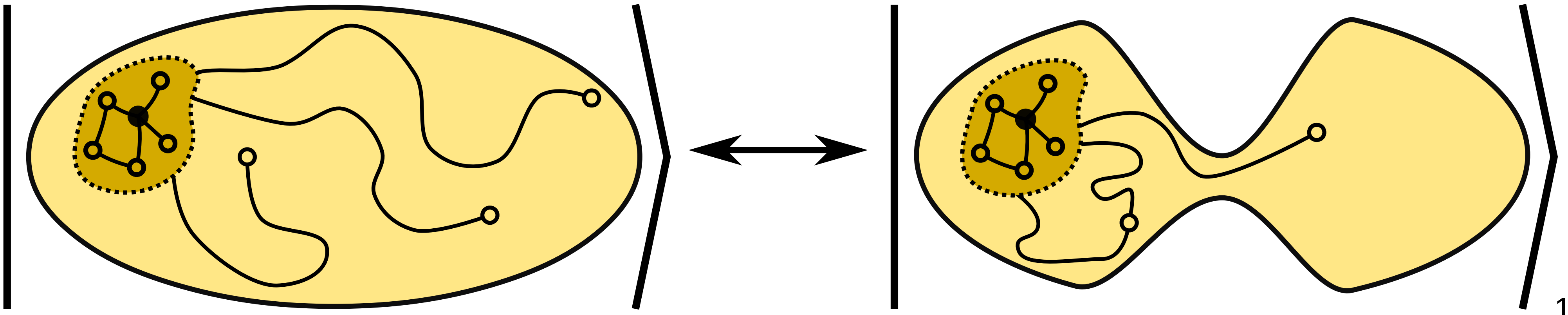
$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$

$$\in O(n^{1/2}t^{1/2})$$

$$\mathcal{S} \subset \mathcal{V},\ |\mathcal{S}| = n^{1/3}:$$

$$\|P^t|\mathcal{S}\rangle\| \geq |\langle\pi|\mathcal{S}\rangle| \geq n^{-1/3}$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



Expansion Testing

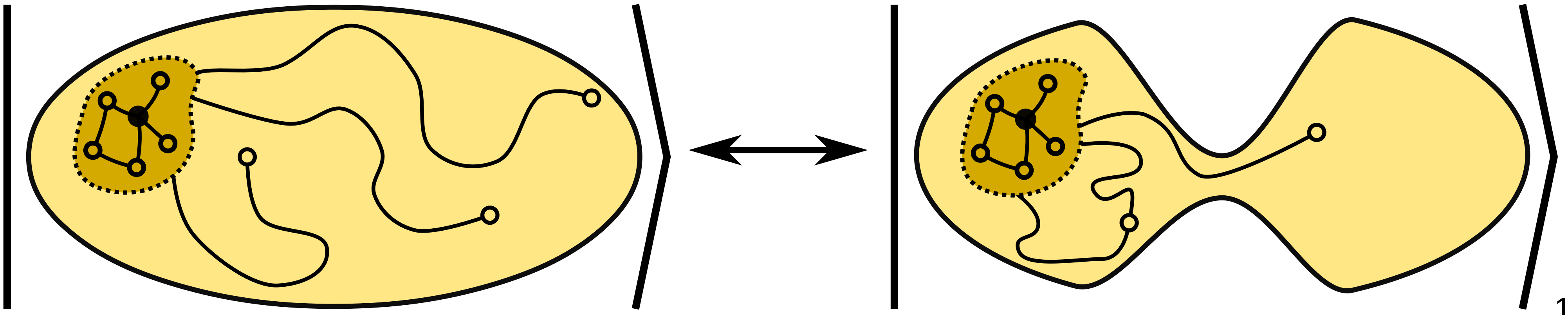
quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/3}t^{1/2})$$

$$\mathcal{S} \subset \mathcal{V},\ |\mathcal{S}| = n^{1/3}:$$

$$\|P^t|\mathcal{S}\rangle\| \geq |\langle\pi|\mathcal{S}\rangle| \geq n^{-1/3}$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



Expansion Testing

? optimal dependency on n

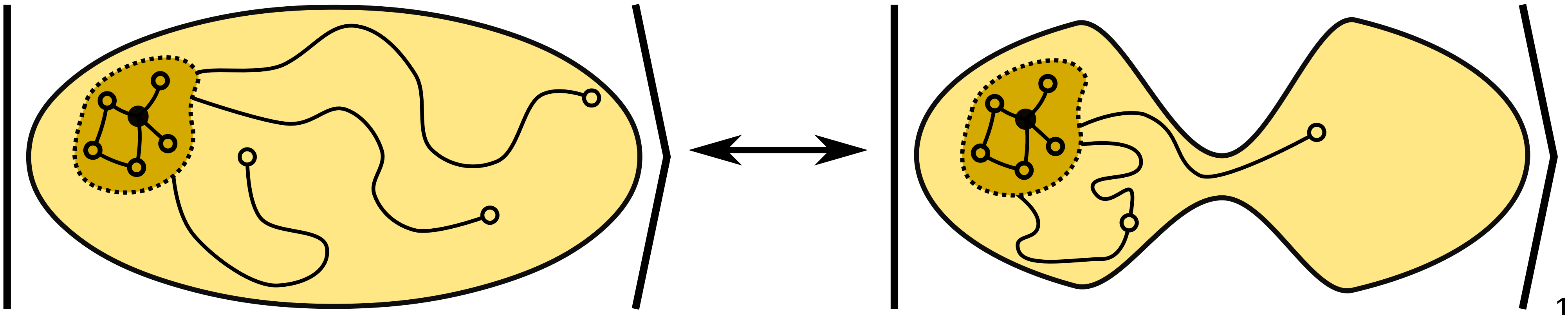
quantum expansion tester:

$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/3}t^{1/2})$$

$\mathcal{S} \subset \mathcal{V},\ |\mathcal{S}| = n^{1/3}$:

$$\|P^t|\mathcal{S}\rangle\| \geq |\langle\pi|\mathcal{S}\rangle| \geq n^{-1/3}$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



Expansion Testing

? optimal dependency on n

quantum expansion tester:

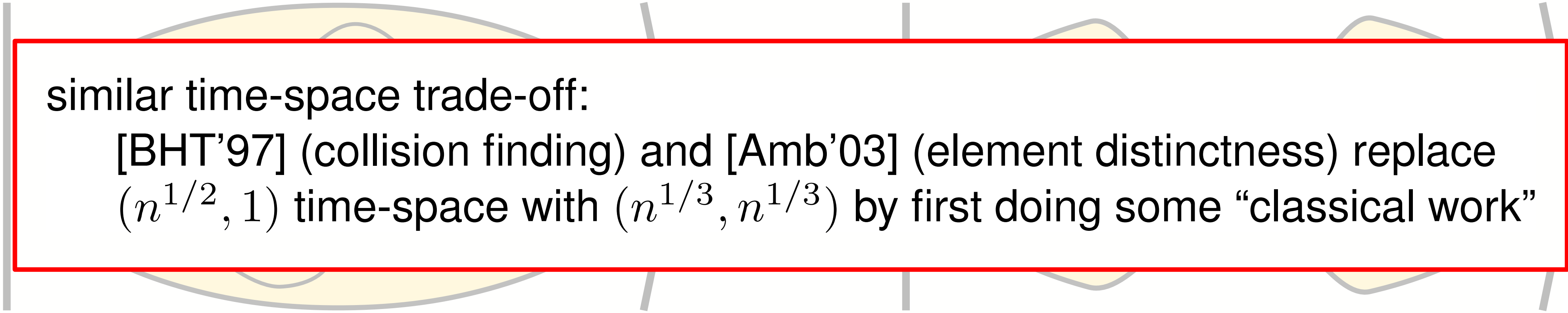
$$O(\|P^t|v\rangle\|^{-1}t^{1/2})$$

$$\in O(n^{1/3}t^{1/2})$$

$\mathcal{S} \subset \mathcal{V},\ |\mathcal{S}| = n^{1/3}$:

$$\|P^t|\mathcal{S}\rangle\| \geq |\langle\pi|\mathcal{S}\rangle| \geq n^{-1/3}$$

improve projection on $|\pi\rangle$
by (classically) growing seed set from v



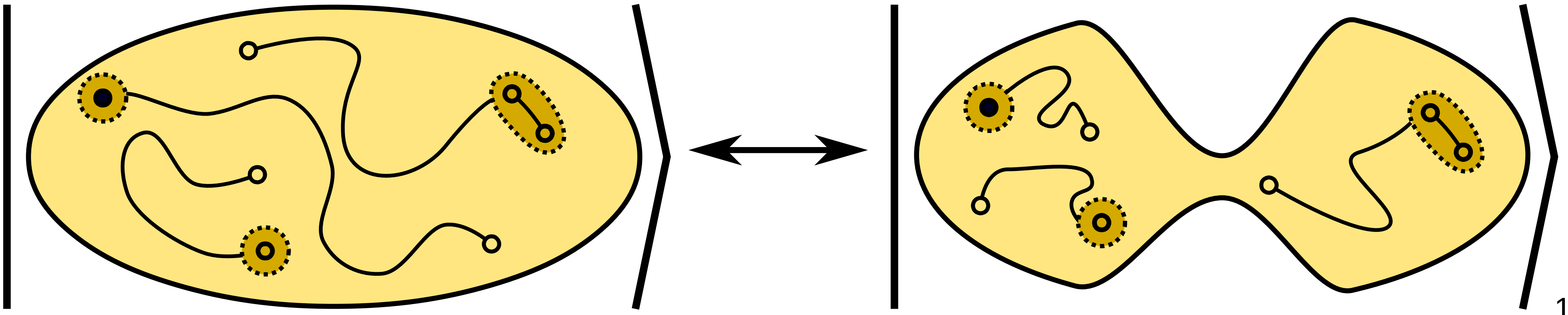
similar time-space trade-off:

[BHT'97] (collision finding) and [Amb'03] (element distinctness) replace $(n^{1/2}, 1)$ time-space with $(n^{1/3}, n^{1/3})$ by first doing some "classical work"

How to Grow a Seed Set?

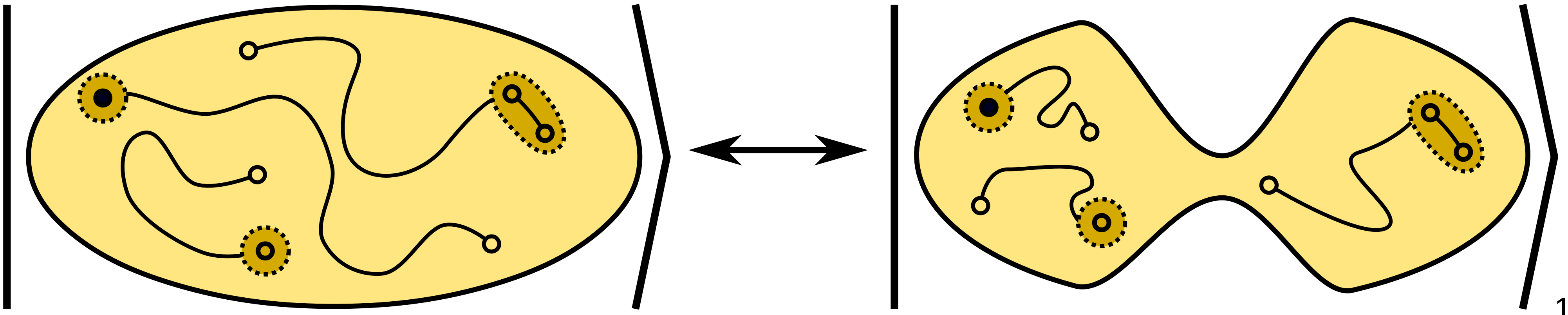
How to Grow a Seed Set?

◇ randomly select $n^{1/3}$ nodes



How to Grow a Seed Set?

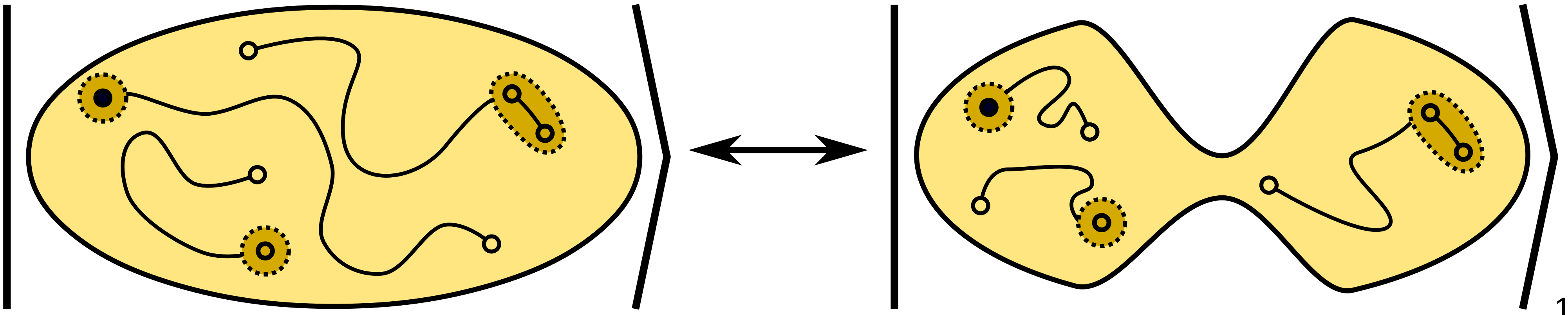
- ◇ randomly select $n^{1/3}$ nodes ↯ balanced w.h.p.!



How to Grow a Seed Set?

◇ ~~randomly select $n^{1/3}$ nodes~~ ↯ balanced w.h.p.!

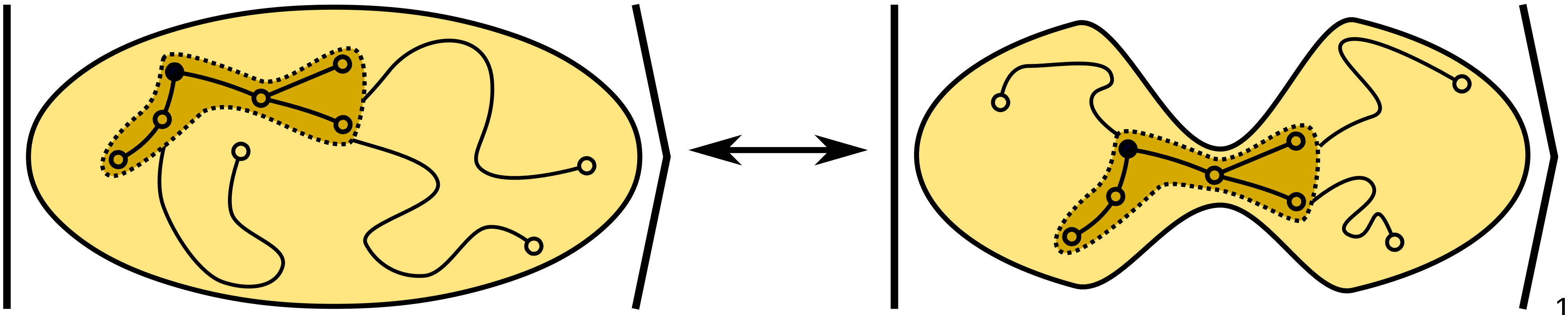
$P^t|\mathcal{S}\rangle \approx \pi$ $P^t|S\rangle \approx \pi$



How to Grow a Seed Set?

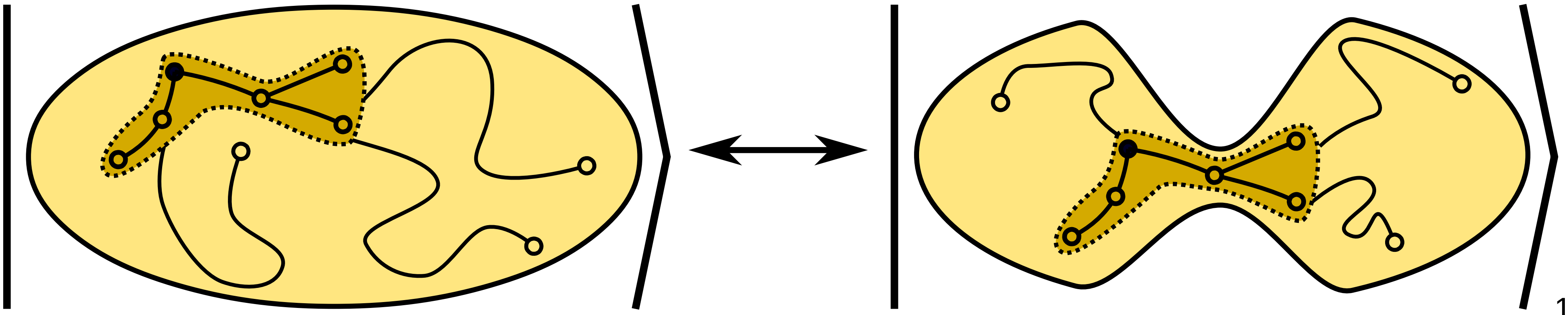
◇ ~~randomly select $n^{1/3}$ nodes~~ ↯ balanced w.h.p.!

◇ use DFS/BFS



How to Grow a Seed Set?

- ◇ ~~randomly select $n^{1/3}$ nodes~~ ↯ balanced w.h.p.!
- ◇ use DFS/BFS ? balanced w.h.p. $\rightarrow$ open question



How to Grow a Seed Set?

◇ ~~randomly select $n^{1/3}$ nodes~~ ↯ balanced w.h.p.!

◇ use DFS/BFS ? balanced w.h.p. $\to$ open question

does suffice for sampling $|\pi\rangle$! [arXiv:1904.11446]

- folklore (using [MNRS'06]): $O(n^{1/2}\delta^{-1/2})$ QW steps
- using seed sets: $O(n^{1/3}\delta^{-1/3})$ QW steps

e.g., creating superposition over graph isomorphisms/black box group in $O(n^{1/3})$

How to Grow a Seed Set?

◇ use tool from local graph clustering!

How to Grow a Seed Set?

◇ use tool from local graph clustering!

“evolving set process”

[Andersen-Oveis Gharan-Peres-Trevisan’12]

How to Grow a Seed Set?

◇ use tool from local graph clustering!

“evolving set process”

[Andersen-Oveis Gharan-Peres-Trevisan’12]

= Markov chain on node subsets:

How to Grow a Seed Set?

$\diamond$ use tool from local graph clustering!

“evolving set process”

[Andersen-Oveis Gharan-Peres-Trevisan’12]

= Markov chain on node subsets: from $\mathcal{S} \subseteq \mathcal{V}$

- pick $Z \in [0,1]$ u.a.r.
- $\forall u \in \mathcal{V} : u \in \mathcal{S}'$ iff $|E(u,\mathcal{S})|/d(u) \geq Z$

How to Grow a Seed Set?

$\diamond$ use tool from local graph clustering!

“evolving set process”

[Andersen-Oveis Gharan-Peres-Trevisan’12]

= Markov chain on node subsets: from $\mathcal{S} \subseteq \mathcal{V}$

- pick $Z \in [0, 1]$ u.a.r.
- $\forall u \in \mathcal{V} : u \in \mathcal{S}'$ iff $|E(u, \mathcal{S})|/d(u) \geq Z$

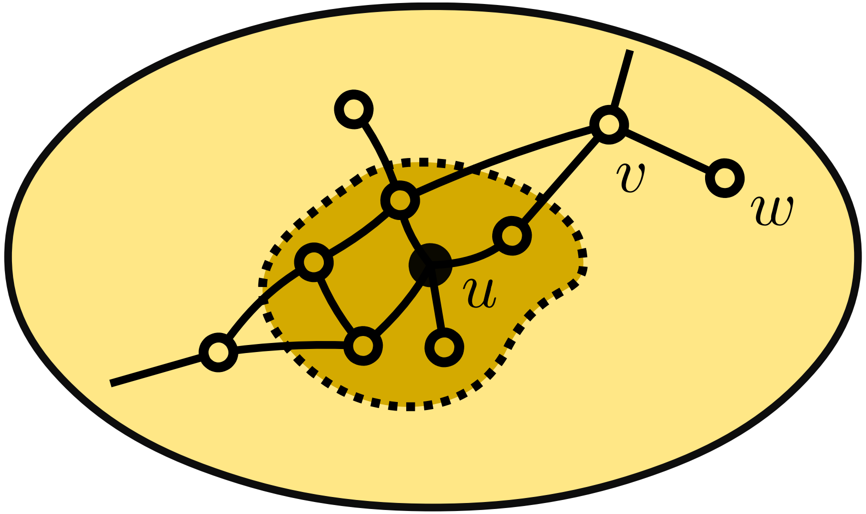
only boundary $\partial\mathcal{S}$ changes

How to Grow a Seed Set?

◇ use tool from local graph clustering!

"evolving set process"

[Andersen-Oveis Gharan-Peres-Trevisan'12]



from $\mathcal{S} \subseteq \mathcal{V}$

- pick $Z \in [0,1]$ u.a.r.
- $\forall u \in \mathcal{V} : u \in \mathcal{S}'$ iff $|E(u,\mathcal{S})|/d(u) \geq Z$

only boundary $\partial\mathcal{S}$ changes

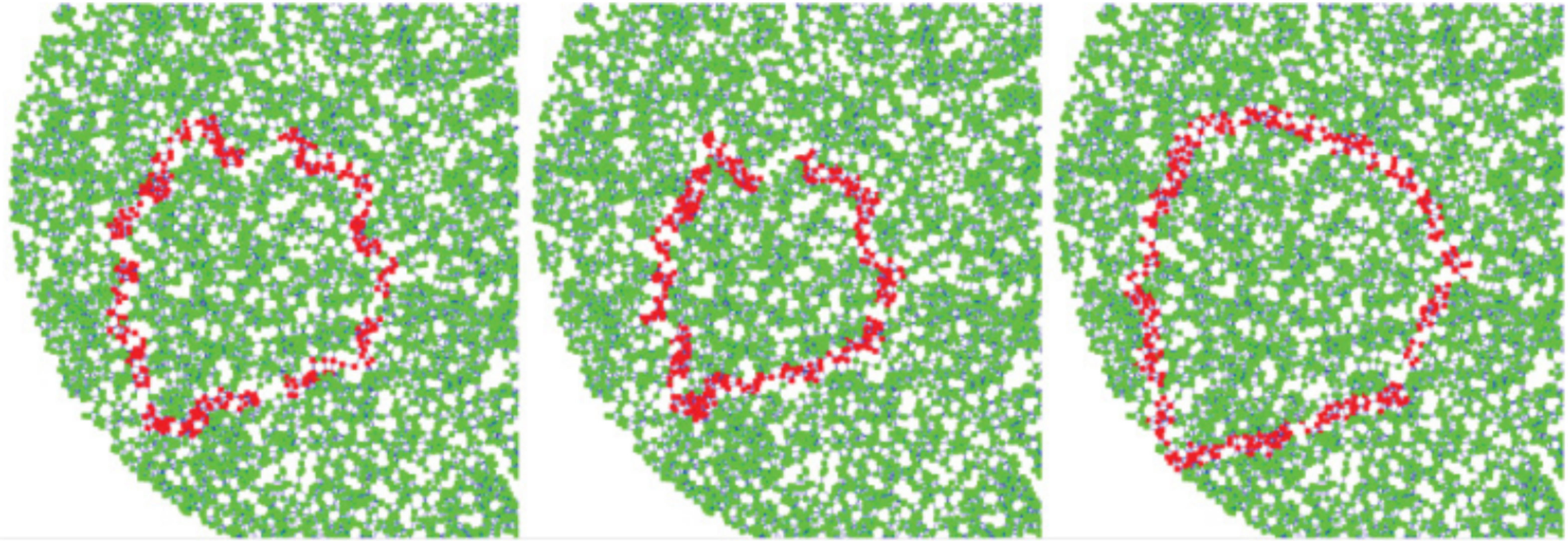
$|E(u,\mathcal{S})|/d(u) = 1, \ |E(v,\mathcal{S})|/d(v) = 1/2, \ |E(w,\mathcal{S})|/d(w) = 0$

How to Grow a Seed Set?

◇ use tool from local graph clustering!

“evolving set process”

[Andersen-Oveis Gharan-Peres-Trevisan’12]



232 steps 235 steps 273 steps

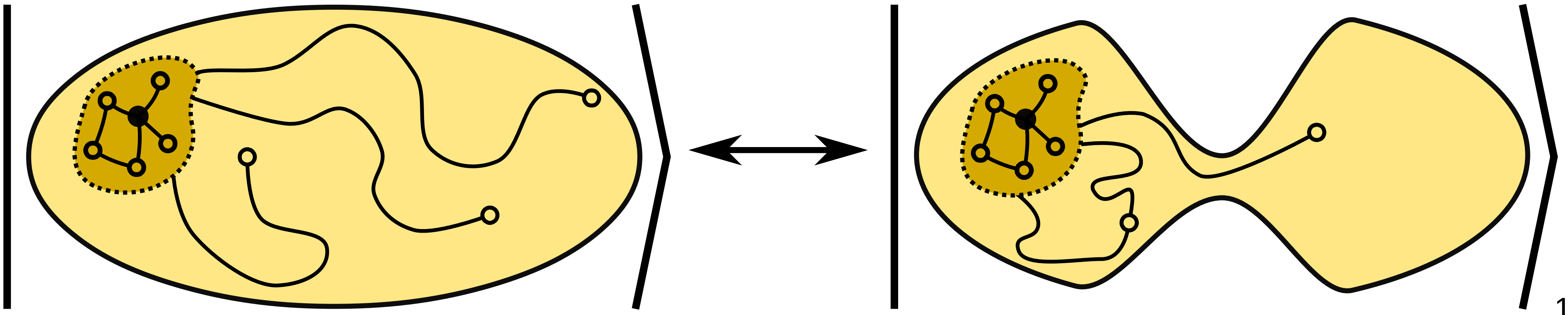
How to Grow a Seed Set?

◇ use tool from local graph clustering!

"evolving set process"

[Andersen-Oveis Gharan-Peres-Trevisan'12]

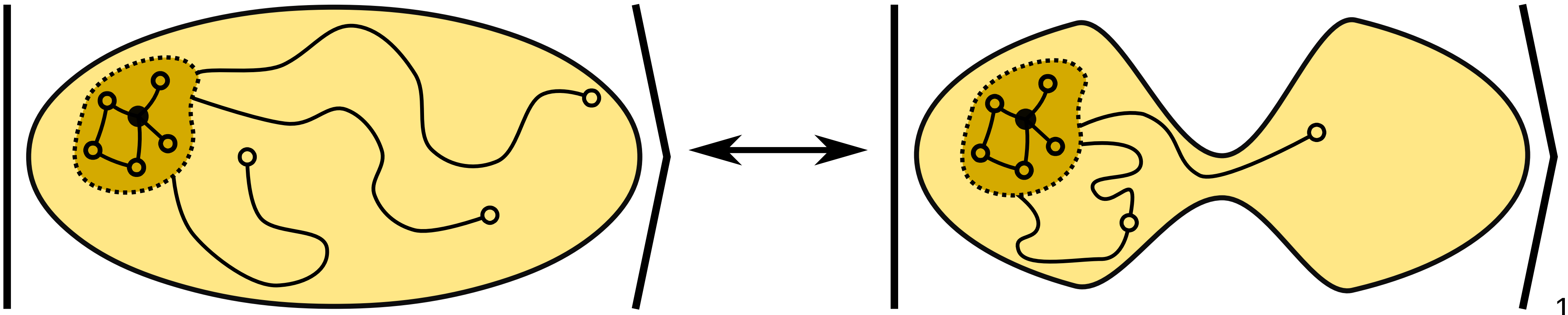
prop.: ESP returns a set of size $n^{1/3}$ within cluster in $O(n^{1/3}\Upsilon^{-1})$ steps



Expansion Testing

quantum expansion tester:

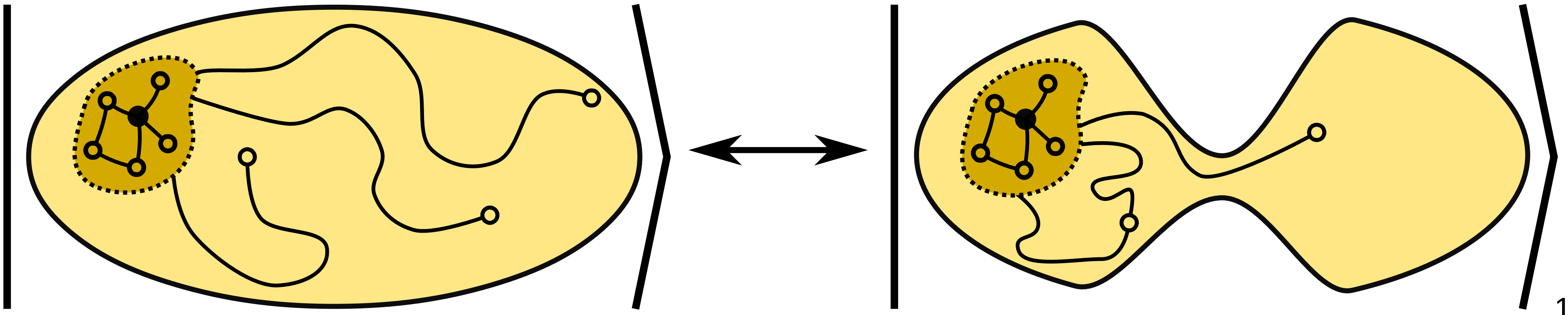
- pick uniformly random node v
- use QFF to create $P^t|v\rangle + |\Gamma\rangle$
- estimate $\|P^t|v\rangle\|$ via ampl. estimation



Expansion Testing

quantum expansion tester:

- pick uniformly random node v
- use ESP to create seed set $\mathcal{S}$, $|\mathcal{S}| = n^{1/3}$
- use QFF to create $P^t|\mathcal{S}\rangle|0\rangle + |\Gamma\rangle$
- estimate $\|P^t|\mathcal{S}\rangle\|$ via ampl. estimation

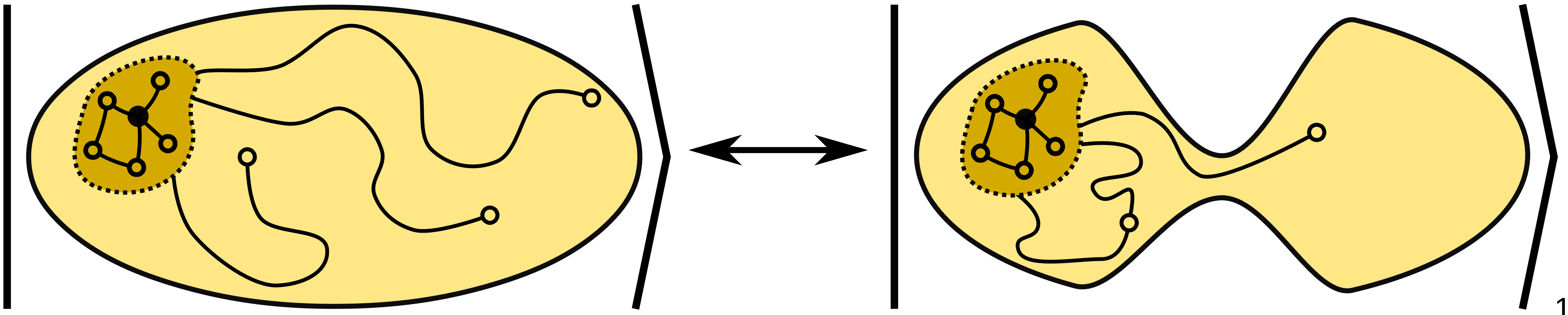


Expansion Testing

quantum expansion tester:

$$O(|\mathcal{S}|\Upsilon^{-1} + \|P^t|\mathcal{S}\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/3}\Upsilon^{-1})$$

- pick uniformly random node v
- use ESP to create seed set $\mathcal{S}$, $|\mathcal{S}| = n^{1/3}$
- use QFF to create $P^t|\mathcal{S}\rangle|0\rangle + |\Gamma\rangle$
- estimate $\|P^t|\mathcal{S}\rangle\|$ via ampl. estimation



Expansion Testing

quantum expansion tester:

$$O(|\mathcal{S}|\Upsilon^{-1} + \|P^t|\mathcal{S}\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/3}\Upsilon^{-1})$$

- pick uniformly random node v
- use ESP to create seed set $\mathcal{S}$, $|\mathcal{S}| = n^{1/3}$
- use QFF to create $P^t|\mathcal{S}\rangle|0\rangle + |\Gamma\rangle$
- estimate $\|P^t|\mathcal{S}\rangle\|$ via ampl. estimation

[Goldreich-Ron ’00]	$O(n^{1/2}\Upsilon^{-2})$ (conj.)	RW collision counting
[CS ’07], [KS ’07], [NS ’07]	$O(n^{1/2}\Upsilon^{-2})$	
[Ambainis-Childs-Liu ’10]	$O(n^{1/3}\Upsilon^{-2})$ (q)	element distinctness
[A-Sarlette ’18]	$O(n^{1/2}\Upsilon^{-1})$ (q)	QFF
[A ’19]	$O(n^{1/3}\Upsilon^{-1})$ (q)	QFF and seed sets

Expansion Testing

- pick uniformly random node v
- use ESP to create seed set $\mathcal{S}$, $|\mathcal{S}| = n^{1/3}$
- use QFF to create $P^t|\mathcal{S}\rangle|0\rangle + |\Gamma\rangle$
- estimate $\|P^t|\mathcal{S}\rangle\|$ via ampl. estimation

quantum expansion tester:

$$O(|\mathcal{S}|\Upsilon^{-1} + \|P^t|\mathcal{S}\rangle\|^{-1}t^{1/2})$$
$$\in O(n^{1/3}\Upsilon^{-1})$$

recent work on testing graph clusterability: [Czumaj et al '15], [Chiplunkar et al '18]

"can graph be appropriately partitioned into k clusters?"

should allow for same speedup using similar ideas!

Open Questions

- expansion testing:

Open Questions

- expansion testing:
  - ◇ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!

Open Questions

- expansion testing:
  - ◇ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!
  - ◇ can we use ESP to speed up classical expansion testing?

Open Questions

- expansion testing:
  - ◇ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!
  - ◇ can we use ESP to speed up classical expansion testing?
  - ◇ close gap: upper bound $O(n^{1/3}\Upsilon^{-1})$, lower bound $\Omega(n^{1/4})$ [ACL'10]

Open Questions

- expansion testing:
  - $\diamond$ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!
  - $\diamond$ can we use ESP to speed up classical expansion testing?
  - $\diamond$ close gap: upper bound $O(n^{1/3}\Upsilon^{-1})$, lower bound $\Omega(n^{1/4})$ [ACL'10]
- use techniques to speed up max/min/sparsest cut approximation algorithms?

Open Questions

- expansion testing:
  - $\diamond$ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!
  - $\diamond$ can we use ESP to speed up classical expansion testing?
  - $\diamond$ close gap: upper bound $O(n^{1/3}\Upsilon^{-1})$, lower bound $\Omega(n^{1/4})$ [ACL'10]
- use techniques to speed up max/min/sparsest cut approximation algorithms?
- QFF + ampl.ampl.: generate $P^t|v\rangle/\|P^t|v\rangle\|$ in $O(t^{1/2}\,n^{1/2})$ steps

  $\rightarrow$ general speedup to $O(t^{1/2}\,n^{1/3})$? $O(t^{1/3}\,n^{1/3})$?

Open Questions

- expansion testing:
  - $\diamond$ can we use simpler BFS instead of ESP? if yes, $O(n^{1/3}\Upsilon^{-2/3})$ tester!
  - $\diamond$ can we use ESP to speed up classical expansion testing?
  - $\diamond$ close gap: upper bound $O(n^{1/3}\Upsilon^{-1})$, lower bound $\Omega(n^{1/4})$ [ACL'10]
- use techniques to speed up max/min/sparsest cut approximation algorithms?
- QFF + ampl.ampl.: generate $P^t|v\rangle/\|P^t|v\rangle\|$ in $O(t^{1/2}\,n^{1/2})$ steps

  $\rightarrow$ general speedup to $O(t^{1/2}\,n^{1/3})$? $O(t^{1/3}\,n^{1/3})$?
- improved ground state preparation for general Markov chains / Hamiltonians using seed sets?